

Ejercicio 1

a) $P(x) = -3x^4 + 5x^3 - 2x^2 + 7x - 6$ para $x = -2$

$$P(-2) = -3 \cdot (-2)^4 + 5 \cdot (-2)^3 - 2 \cdot (-2)^2 + 7 \cdot (-2) - 6 = -48 - 40 - 8 - 14 - 6 = -116 \Rightarrow P(-2) = -116$$

b) $P(x, y) = 3xy - 2x^2 + 3y - 1$ para $x = -2$ e $y = -3$

$$P(-2, -3) = 3 \cdot (-2) \cdot (-3) - 2 \cdot (-2)^2 + 3 \cdot (-3) - 1 = 18 - 8 - 9 - 1 = 0 \Rightarrow P(-2, -3) = 0$$

Ejercicio 2

$$\begin{aligned} 1) \quad 2A(x) - 3B(x) - D(x) &= 2 \cdot (-5x^4 + 2x^3 - 3x^2 - 8x - 3) - 3 \cdot (3x^4 - 2x^3 + 5x^2 - 2) - (x^3 + 3x^2 - 4x + 2) = \\ &= -10x^4 + 4x^3 - 6x^2 - 16x - 6 - 9x^4 + 6x^3 - 15x^2 + 6 - x^3 - 3x^2 + 4x - 2 = \\ &= -19x^4 + 9x^3 - 24x^2 - 12x - 2 \end{aligned}$$

$$\begin{aligned} 2) \quad 2 \cdot C(x) - [3 \cdot D(x) - 4 \cdot E(x)] &= \\ &= 2 \cdot (x^3 - 2x^2 + 5) - [3 \cdot (x^3 + 3x^2 - 4x + 2) - 4 \cdot (3x^2 - 2x + 4)] = \\ &= (2x^3 - 4x^2 + 10) - (3x^3 + 9x^2 - 12x + 6 - 12x^2 + 8x - 16) = \\ &= (2x^3 - 4x^2 + 10) - (3x^3 - 3x^2 - 4x - 10) = \\ &= 2x^3 - 4x^2 + 10 - 3x^3 + 3x^2 + 4x + 10 = \\ &= -x^3 - x^2 + 4x + 20 \end{aligned}$$

$$\begin{aligned} 3) \quad C(x) \cdot E(x) &= (x^3 - 2x^2 + 5) \cdot (3x^2 - 2x + 4) = (\text{Propiedad distributiva "todo } \times \text{ todo"}) \\ &= 3x^5 - 2x^4 + 4x^3 - 6x^4 + 4x^3 - 8x^2 + 15x^2 - 10x + 20 = \\ &= 3x^5 - 8x^4 + 8x^3 + 7x^2 - 10x + 20 \end{aligned}$$

$$\begin{aligned} 4) \quad D(x) \cdot E(x) &= (x^3 + 3x^2 - 4x + 2) \cdot (3x^2 - 2x + 4) = (\text{Propiedad distributiva}) \\ &= 3x^5 - 2x^4 + 4x^3 + 9x^4 - 6x^3 + 12x^2 - 12x^3 + 8x^2 - 16x + 6x^2 - 4x + 8 = \\ &= 3x^5 + 7x^4 - 14x^3 + 26x^2 - 20x + 8 \end{aligned}$$

$$\begin{aligned} 5) \quad [C(x) - D(x)] \cdot E(x) &= [(x^3 - 2x^2 + 5) - (x^3 + 3x^2 - 4x + 2)] \cdot (3x^2 - 2x + 4) = \\ &= (x^3 - 2x^2 + 5 - x^3 - 3x^2 + 4x - 2) \cdot (3x^2 - 2x + 4) = \\ &= (-5x^2 + 4x + 3) \cdot (3x^2 - 2x + 4) = (\text{Propiedad distributiva}) \\ &= -15x^4 + 10x^3 - 20x^2 + 12x^3 - 8x^2 + 16x + 9x^2 - 6x + 12 = \\ &= -15x^4 + 22x^3 - 19x^2 + 10x + 12 \end{aligned}$$

$$\begin{aligned}
6) \quad C^2(x) &= C(x) \cdot C(x) = (x^3 - 2x^2 + 5) \cdot (x^3 - 2x^2 + 5) = \\
&= x^3 \cdot (x^3 - 2x^2 + 5) - 2x^2 \cdot (x^3 - 2x^2 + 5) + 5 \cdot (x^3 - 2x^2 + 5) = \\
&= x^6 - 2x^5 + 5x^3 - 2x^5 + 4x^4 - 10x^2 + 5x^3 - 10x^2 + 25 = \\
&= \boxed{x^6 - 4x^5 + 4x^4 + 10x^3 - 20x^2 + 25}
\end{aligned}$$

Ejercicio 3

$$\begin{aligned}
1) \quad &4x \cdot (x^2 - 2x + 6) - (x - 1) \cdot (2 - x^2) - 4x^2 \cdot (6x + 3) = \\
&= (4x^3 - 8x^2 + 24x) - (2x - x^3 - 2 + x^2) - (24x^3 + 12x^2) = \\
&= 4x^3 - 8x^2 + 24x - 2x + x^3 + 2 - x^2 - 24x^3 - 12x^2 = \\
&= \boxed{-19x^3 - 21x^2 + 22x + 2}
\end{aligned}$$

$$\begin{aligned}
2) \quad &-2x^2 - 3x \cdot (-5 + 2x) + 2x^3 - (1 - x) \cdot (-x^2 + 3) = \\
&= -2x^2 - (-15x + 6x^2) + 2x^3 - (-x^2 + 3 + x^3 - 3x) = \\
&= -2x^2 + 15x - 6x^2 + 2x^3 + x^2 - 3 - x^3 + 3x = \\
&= \boxed{x^3 - 7x^2 + 18x - 3}
\end{aligned}$$

$$\begin{aligned}
3) \quad &-3x^2 \cdot (3x - 5) - 2 \cdot (x^3 - 5x + 3) - (2 - x^2) \cdot (2x - 1) = \\
&= (-9x^3 + 15x^2) - (2x^3 - 10x + 6) - (4x - 2 - 2x^3 + x^2) = \\
&= -9x^3 + 15x^2 - 2x^3 + 10x - 6 - 4x + 2 + 2x^3 - x^2 = \\
&= \boxed{-9x^3 + 14x^2 + 6x - 4}
\end{aligned}$$

$$\begin{aligned}
4) \quad &4a^2 - (3a^2 - 5a) \cdot (-3 + 2a) - (1 - 4a) \cdot (2a^2 - 5) = \\
&= 4a^2 - (-9a^2 + 6a^3 + 15a - 10a^2) - (2a^2 - 5 - 8a^3 + 20a) = \\
&= 4a^2 + 9a^2 - 6a^3 - 15a + 10a^2 - 2a^2 + 5 + 8a^3 - 20a = \\
&= \boxed{2a^3 + 21a^2 - 35a + 5}
\end{aligned}$$

$$\begin{aligned}
5) \quad &15x - (4x - 1) \cdot (x^2 + 3x) - (2x - 7) \cdot (-6 + x^2) = \\
&= 15x - (4x^3 + 12x^2 - x^2 - 3x) - (-12x + 2x^3 + 42 - 7x^2) = \\
&= 15x - 4x^3 - 12x^2 + x^2 + 3x + 12x - 2x^3 - 42 + 7x^2 = \\
&= \boxed{-6x^3 - 4x^2 + 30x - 42}
\end{aligned}$$

- 6) $(x-2) \cdot (2x^2+x-3) - 3x^3 - (-3x^2+4) \cdot (5x-2) =$
 $= (2x^3+x^2-3x-4x^2-2x+6) - 3x^3 - (-15x^3+6x^2+20x-8) =$
 $= 2x^3+x^2-3x-4x^2-2x+6-3x^3+15x^3-6x^2-20x+8 =$
 $= 14x^3-9x^2-25x+14$
- 7) $(x-5x^2) \cdot (1-2x) - (x^2-1) \cdot (5-2x) - (-x^2+3) \cdot (-x+1) =$
 $= (x-2x^2-5x^2+10x^3) - (5x^2-2x^3-5+2x) - (x^3-x^2-3x+3) =$
 $= x-2x^2-5x^2+10x^3-5x^2+2x^3+5-2x-x^3+x^2+3x-3 =$
 $= 11x^3-11x^2+2x+2$
- 8) $3x^2 - (-2x^2+3) \cdot (x+5x^2) - 2x \cdot (-3x^3+4x-2) =$
 $= 3x^2 - (-2x^3-10x^4+3x+15x^2) - (-6x^4+8x^2-4x) =$
 $= 3x^2+2x^3+10x^4-3x-15x^2+6x^4-8x^2+4x =$
 $= 16x^4+2x^3-20x^2+x$
- 9) $-x^2 \cdot (3x-2) - (1-6x) \cdot (3-x^2) - (1+x^2-5x) \cdot (3-x) =$
 $= (-3x^3+2x^2) - (3-x^2-18x+6x^3) - (3-x+3x^2-x^3-15x+5x^2) =$
 $= -3x^3+2x^2-3+x^2+18x-6x^3-3+x-3x^2+x^3+15x-5x^2 =$
 $= -8x^3-5x^2+34x-6$
- 10) $3x^2 \cdot (-x^2+3x-2) - (x^3-3) \cdot (1+x^2) - (5x+4) \cdot (-x^4+2x) =$
 $= (-3x^4+9x^3-6x^2) - (x^3+x^5-3-3x^2) - (-5x^5+10x^2-4x^4+8x) =$
 $= -3x^4+9x^3-6x^2-x^3-x^5+3+3x^2+5x^5-10x^2+4x^4-8x =$
 $= 4x^5+x^4+8x^3-13x^2-8x+3$

Ejercicio 4

1) $(-6x^5 + 7x^4 + 3x^3 + 5x^2 - 4) : (-2x^2 + x + 1) =$

$$\begin{array}{r}
 -6x^5 + 7x^4 + 3x^3 + 5x^2 - 4 \quad \left| \begin{array}{l} -2x^2 + x + 1 \\ \hline 3x^3 - 2x^2 - x - 4 \end{array} \right. \\
 + 6x^5 - 3x^4 - 3x^3 \\
 \hline
 + 4x^4 + 5x^2 - 4 \\
 - 4x^4 + 2x^3 + 2x^2 \\
 \hline
 + 2x^3 + 7x^2 - 4 \\
 - 2x^3 + x^2 + x \\
 \hline
 + 8x^2 + x - 4 \\
 - 8x^2 + 4x + 4 \\
 \hline
 5x
 \end{array}$$

Cociente = $3x^3 - 2x^2 - x - 4$

Resto = $5x$

2) $(2x^4 - x^2 - 3x^3 - 8x - 1) : (-x^2 - x + 2) =$

$$\begin{array}{r}
 2x^4 - 3x^3 - x^2 - 8x - 1 \quad \left| \begin{array}{l} -x^2 - x + 2 \\ \hline -2x^2 + 5x - 8 \end{array} \right. \\
 - 2x^4 - 2x^3 + 4x^2 \\
 \hline
 -5x^3 + 3x^2 - 8x - 1 \\
 + 5x^3 + 5x^2 - 10x \\
 \hline
 + 8x^2 - 18x - 1 \\
 - 8x^2 - 8x + 16 \\
 \hline
 -26x + 15
 \end{array}$$

Cociente = $-2x^2 + 5x - 8$

Resto = $-26x + 15$

3) $(-x^6 + 4x^5 - x^4 + 8x^3 + 4x^2 - 1) : (x^3 - x + 3) =$

$$\begin{array}{r}
 -x^6 + 4x^5 - x^4 + 8x^3 + 4x^2 - 1 \quad \left| \begin{array}{l} x^3 - x + 3 \\ \hline -x^3 + 4x^2 - 2x + 15 \end{array} \right. \\
 + x^6 \\
 \hline
 + 4x^5 - 2x^4 + 11x^3 + 4x^2 - 1 \\
 - 4x^5 + 4x^3 - 12x^2 \\
 \hline
 - 2x^4 + 15x^3 - 8x^2 - 1 \\
 + 2x^4 - 2x^2 + 6x \\
 \hline
 + 15x^3 - 10x^2 + 6x - 1 \\
 - 15x^3 + 15x - 45 \\
 \hline
 -10x^2 + 21x - 46
 \end{array}$$

Cociente = $-x^3 + 4x^2 - 2x + 15$

Resto = $-10x^2 + 21x - 46$

$$4) (x^6 - x^5 + 2x^4 - 3x^3 - 8x - 1) : (x^2 - x - 2) =$$

$$\begin{array}{r}
 x^6 - x^5 + 2x^4 - 3x^3 - 8x - 1 \quad \left| \begin{array}{l} x^2 - x - 2 \\ x^4 + 4x^2 + x + 9 \end{array} \right. \\
 \underline{-x^6 + x^5 + 2x^4} \\
 +4x^4 - 3x^3 - 8x - 1 \\
 \underline{-4x^4 + 4x^3 + 8x^2} \\
 x^3 + 8x^2 - 8x - 1 \\
 \underline{-x^3 + x^2 + 2x} \\
 +9x^2 - 6x - 1 \\
 \underline{-9x^2 + 9x + 18} \\
 +3x + 17
 \end{array}$$

$$\text{Cociente} = x^4 + 4x^2 + x + 9$$

$$\text{Resto} = 3x + 17$$

$$5) (2x^5 + 2x^4 - 3x^2 - 8x - 1) : (-x^2 - 2x + 1) =$$

$$\begin{array}{r}
 2x^5 + 2x^4 - 3x^2 - 8x - 1 \quad \left| \begin{array}{l} -x^2 - 2x + 1 \\ -2x^3 + 2x^2 - 6x + 17 \end{array} \right. \\
 \underline{-2x^5 - 4x^4 + 2x^3} \\
 -2x^4 + 2x^3 - 3x^2 - 8x - 1 \\
 \underline{+2x^4 + 4x^3 - 2x^2} \\
 +6x^3 - 5x^2 - 8x - 1 \\
 \underline{-6x^3 - 12x^2 + 6x} \\
 -17x^2 - 2x - 1 \\
 \underline{+17x^2 + 34x - 17} \\
 +32x - 18
 \end{array}$$

$$\text{Cociente} = -2x^3 + 2x^2 - 6x + 17$$

$$\text{Resto} = 32x - 18$$

$$6) (4x^5 - x^3 - 2x^2 - 1) : (x + 1) =$$

$$\begin{array}{r}
 \begin{array}{cccccc}
 4 & 0 & -1 & -2 & 0 & -1 \\
 -4 & +4 & -3 & +5 & -5 & \\
 \hline
 4 & -4 & +3 & -5 & +5 & -6
 \end{array}
 \end{array}$$

$$\text{Cociente} = 4x^4 - 4x^3 + 3x^2 - 5x + 5$$

$$\text{Resto} = -6$$

7) $(-x^6 + 4x^5 - x^4 + 8x^3 + 4x) : (x - 2) =$

$$\begin{array}{r|rrrrrrr}
 & -1 & +4 & -1 & +8 & 0 & +4 & 0 \\
 +2 & & -2 & +4 & +6 & +28 & +56 & +120 \\
 \hline
 & -1 & +2 & +3 & +14 & +28 & +60 & \boxed{+120}
 \end{array}$$

Cociente = $-x^5 + 2x^4 + 3x^3 + 14x^2 + 28x + 60$ Resto = 120

8) $(2x^5 + 2x^4 - 3x^2 - 8x) : (x + 3) =$

$$\begin{array}{r|rrrrrr}
 & 2 & +2 & 0 & -3 & -8 & 0 \\
 -3 & & -6 & +12 & -36 & +117 & -327 \\
 \hline
 & 2 & -4 & +12 & -39 & +109 & \boxed{-327}
 \end{array}$$

Cociente = $2x^4 - 4x^3 + 12x^2 - 39x + 109$ Resto = -327

9) $(-x^5 - x^4 + 8x^3 + 4x^2 - 1) : (x + 2) =$

$$\begin{array}{r|rrrrrr}
 & -1 & -1 & +8 & +4 & 0 & -1 \\
 -2 & & +2 & -2 & -12 & +16 & -32 \\
 \hline
 & -1 & +1 & +6 & -8 & +16 & \boxed{-33}
 \end{array}$$

Cociente = $-x^4 + x^3 + 6x^2 - 8x + 16$ Resto = -33

10) $(2x^6 - x^5 - 2x^3 + 4x^2 - 3x) : (x - 3) =$

$$\begin{array}{r|rrrrrrr}
 & 2 & -1 & 0 & -2 & +4 & -3 & 0 \\
 +3 & & +6 & +15 & +45 & +129 & +399 & +1188 \\
 \hline
 & 2 & +5 & +15 & +43 & +133 & +396 & \boxed{+1188}
 \end{array}$$

Cociente = $2x^5 + 5x^4 + 15x^3 + 43x^2 + 133x + 396$ Resto = 1188

Ejercicio 5

$$\begin{aligned}
 1) \quad & \underline{(x^2 + 2)} \cdot \underline{(x^2 - 3)} - 2x \cdot \underline{(x - 2)^2} - 3x \cdot \underline{(2 - 4x^2)^2} = \\
 & = (x^4 - 3x^2 + 3x^2 - 6) - 2x \cdot (x^2 - 4x + 4) - 3x \cdot (4 - 16x^2 + 16x^4) = \\
 & = x^4 - 3x^2 + 2x^2 - 6 - 2x^3 + 8x^2 - 8x - 12x + 48x^3 - 48x^5 = \\
 & = \boxed{-48x^5 + x^4 + 46x^3 + 7x^2 - 20x - 6}
 \end{aligned}$$

$$(x - 2)^2 = x^2 - 2 \cdot x \cdot 2 + 2^2 = x^2 - 4x + 4$$

$$(2 - 4x^2)^2 = 2^2 - 2 \cdot 2 \cdot 4x^2 + (4x^2)^2 = 4 - 16x^2 + 16x^4$$

$$\begin{aligned}
 2) \quad & \underline{(1 + 4b^2)} \cdot \underline{(1 - 4b^2)} - \underline{(2b^2 + 1)^2} - \underline{(3b - 1)^2} = & \quad (1 + 4b^2) \cdot (1 - 4b^2) = 1^2 - (4b^2)^2 = 1 - 16b^4 \\
 & = (1 - 16b^4) - (4b^4 + 4b^2 + 1) - (9b^2 - 6b + 1) = & \quad (2b^2 + 1)^2 = (2b^2)^2 + 2 \cdot 2b^2 \cdot 1 + 1^2 = 4b^4 + 4b^2 + 1 \\
 & = 1 - 16b^4 - 4b^4 - 4b^2 - 1 - 9b^2 + 6b - 1 = & \quad (3b - 1)^2 = (3b)^2 - 2 \cdot 3b \cdot 1 + 1^2 = 9b^2 - 6b + 1 \\
 & = \boxed{-20b^4 - 13b^2 + 6b - 1}
 \end{aligned}$$

$$\begin{aligned}
 3) \quad & \underline{(3x^3 + 2)^2} - \underline{(x^3 + 7) \cdot (x^3 - 7)} - 3x \cdot \underline{(5x^2 - 3x)^2} = \\
 & = (9x^6 + 12x^3 + 4) - (x^6 - 49) - 3x \cdot (25x^4 - 30x^3 + 9x^2) = \\
 & = 9x^6 + 12x^3 + 4 - x^6 + 49 - 75x^5 + 90x^4 - 27x^3 = \\
 & = \boxed{8x^6 - 75x^5 + 90x^4 - 15x^3 + 53}
 \end{aligned}$$

$$(3x^3 + 2)^2 = (3x^3)^2 + 2 \cdot 3x^3 \cdot 2 + 2^2 = 9x^6 + 12x^3 + 4$$

$$(x^3 + 7) \cdot (x^3 - 7) = (x^3)^2 - 7^2 = x^6 - 49$$

$$(5x^2 - 3x)^2 = (5x^2)^2 - 2 \cdot 5x^2 \cdot 3x + (3x)^2 = 25x^4 - 30x^3 + 9x^2$$

$$\begin{aligned}
 4) \quad & \underline{(2x + 1) \cdot (2x - 1)} - \underline{(2x^3 - 2)^2} - \underline{(3x^4 - 2x + 1) \cdot (-2x^2 + 4)} = \\
 & = (4x^2 - 1) - (4x^6 - 8x^3 + 4) - (-6x^6 + 12x^4 + 4x^3 - 8x - 2x^2 + 4) = \\
 & = 4x^2 - 1 - 4x^6 + 8x^3 - 4 + 6x^6 - 12x^4 - 4x^3 + 8x + 2x^2 - 4 = \\
 & = \boxed{2x^6 - 12x^4 + 4x^3 + 6x^2 + 8x - 9}
 \end{aligned}$$

$$(2x + 1) \cdot (2x - 1) = (2x)^2 - 1^2 = 4x^2 - 1$$

$$(2x^3 - 2)^2 = (2x^3)^2 - 2 \cdot 2x^3 \cdot 2 + 2^2 = 4x^6 - 8x^3 + 4$$

$$\begin{aligned}
 5) \quad & 5 \cdot (y+1)^2 - 3y - (4y+5) \cdot (4y-5) - (2y-1) \cdot (4y+1) = \\
 & = 5 \cdot (y^2 + 2y + 1) - 3y - (16y^2 - 25) - (8y^2 + 2y - 4y - 1) = \\
 & = 5y^2 + 10y + 5 - 3y - 16y^2 + 25 - 8y^2 - 2y + 4y + 1 = \\
 & = \boxed{-19y^2 + 9y + 31}
 \end{aligned}$$

$$(y+1)^2 = y^2 + 2 \cdot y \cdot 1 + 1^2 = y^2 + 2y + 1$$

$$(4y+5) \cdot (4y-5) = (4y)^2 - 5^2 = 16y^2 - 25$$

$$\begin{aligned}
 6) \quad & 5x^3 - 3x \cdot (1+4x)^2 - (2x+3) \cdot (-x^2 - 5x + 2) = \\
 & = 5x^3 - 3x \cdot (1 + 8x + 16x^2) - (-2x^3 - 10x^2 + 4x - 3x^2 - 15x + 6) = \\
 & = 5x^3 - 3x - 24x^2 - 48x^3 + 2x^3 + 10x^2 - 4x + 3x^2 + 15x - 6 = \\
 & = \boxed{-41x^3 - 11x^2 + 8x - 6}
 \end{aligned}$$

$$(1+4x)^2 = 1^2 + 2 \cdot 4x \cdot 1 + (4x)^2 = 1 + 8x + 16x^2$$

$$\begin{aligned}
 7) \quad & (2-3x^3)^2 - (5-2x) \cdot (5+2x) - (2x^4-3x) \cdot (-3x^2+1) = \\
 & = (4 - 12x^3 + 9x^6) - (25 - 4x^2) - (-6x^6 + 2x^4 + 9x^3 - 3x) = \\
 & = 4 - 12x^3 + 9x^6 - 25 + 4x^2 + 6x^6 - 2x^4 - 9x^3 + 3x = \\
 & = \boxed{15x^6 - 2x^4 - 21x^3 + 4x^2 + 3x - 21}
 \end{aligned}$$

$$(2-3x^3)^2 = 2^2 - 2 \cdot 2x^3 \cdot 2 + (2x^3)^2 = 4 - 8x^3 + 4x^6$$

$$(5-2x) \cdot (5+2x) = 5^2 - (2x)^2 = 25 - 4x^2$$

$$\begin{aligned}
 8) \quad & (3-2x^2)^2 - (2+5x^3) \cdot (2-5x^3) + 5x - x \cdot (x+3)^2 = \\
 & = (9 - 12x^2 + 4x^4) - (4 - 25x^6) + 5x - x \cdot (x^2 + 6x + 9) = \\
 & = 9 - 12x^2 + 4x^4 - 4 + 25x^6 + 5x - x^3 - 6x^2 - 9x = \\
 & = \boxed{25x^6 + 4x^4 - x^3 - 18x^2 - 4x + 5}
 \end{aligned}$$

$$(3-2x^2)^2 = 3^2 - 2 \cdot 3 \cdot 2x^2 + (2x^2)^2 = 9 - 12x^2 + 4x^4$$

$$(2+5x^3) \cdot (2-5x^3) = 2^2 - (5x^3)^2 = 4 - 25x^6$$

$$(x+3)^2 = x^2 + 2 \cdot x \cdot 3 + 3^2 = x^2 + 6x + 9$$

$$\begin{aligned} 9) & \quad (2x+1) \cdot (2x-1) - (2x^3-2)^2 - (3x^4-2x) \cdot (-2x^2+4) = \\ & \quad = (4x^2-1) - 84x^6 - 8x^3 + 4 - (-6x^6 + 12x^4 + 4x^3 - 8x) = \\ & \quad = 4x^2 - 1 - 4x^6 + 8x^3 - 4 + 6x^6 - 12x^4 - 4x^3 + 8x = \\ & \quad = \boxed{2x^6 - 12x^4 + 4x^3 + 4x^2 + 8x - 5} \end{aligned}$$

$$(2x+1) \cdot (2x-1) = (2x)^2 - 1^2 = 4x^2 - 1$$

$$(2x^3-2)^2 = (2x^3)^2 - 2 \cdot 2x^3 \cdot 2 + 2^2 = 4x^6 - 8x^3 + 4$$

$$\begin{aligned} 10) & \quad (\sqrt{2}x^2+1) \cdot (\sqrt{2}x^2-1) - (2x^3-2)^2 - (3x^4-2x) \cdot (-2x^2+4) = \\ & \quad = (2x^4-1) - (4x^6-8x^3+4) - (-6x^6+12x^4+4x^3-8x) = \\ & \quad = 2x^4-1-4x^6+8x^3-4+6x^6-12x^4-4x^3+8x = \\ & \quad = \boxed{2x^6-10x^4+4x^3+8x-5} \end{aligned}$$

Ejercicio 6

Determina el valor de m para que el polinomio $P(x) = 5x^4 - 7x^3 + 2x^2 + 4x + m$ tenga por resto 130 al dividirlo por $(x + 2)$.

$$P(x) : (x + 2) \text{ tiene resto } 130 \Rightarrow P(-2) = 130 \Rightarrow 5 \cdot (-2)^4 - 7 \cdot (-2)^3 + 2 \cdot (-2)^2 + 4 \cdot (-2) + m = 130 \Rightarrow$$

Teorema del resto

$$\Rightarrow 80 + 56 + 8 - 8 + m = 130 \Rightarrow 136 + m = 130 \Rightarrow m = -6$$

Ejercicio 7

Un polinomio de segundo grado tiene por coeficiente principal 1, se anula para $x = 3$ y toma el valor 4 para $x = 5$. Hállalo.

$$P(x) \text{ polinomio de } 2^\circ \text{ grado} \Rightarrow P(x) = ax^2 + bx + c$$

- ♦ El coeficiente principal es 1 $\Rightarrow a = 1 \Rightarrow P(x) = x^2 + bx + c$
- ♦ $P(x)$ se anula en $x = 3 \Rightarrow P(3) = 0 \Rightarrow (3)^2 + b \cdot (3) + c = 0 \Rightarrow 9 + 3b + c = 0 \Rightarrow 3b + c = -9$
- ♦ $P(x)$ toma el valor 4 en $x = 5 \Rightarrow P(5) = 4 \Rightarrow 5^2 + b \cdot 5 + c = 4 \Rightarrow 25 + 5b + c = 4 \Rightarrow 5b + c = -21$

Luego

$$\begin{cases} 3b + c = -9 \\ 5b + c = -21 \end{cases} \xrightarrow{\cdot(-1)} \begin{cases} -3b - c = 9 \\ 5b + c = -21 \end{cases}$$

$$2b = -12 \Rightarrow b = \frac{-12}{2} \Rightarrow b = -6$$

$$\begin{cases} 3b + c = -9 \\ b = -6 \end{cases} \Rightarrow 3 \cdot (-6) + c = -9 \Rightarrow -18 + c = -9 \Rightarrow c = 9$$

Por tanto, $P(x) = x^2 - 6x + 9$

Ejercicio 8

Halla un polinomio de primer grado que dividido por $(x - 1)$ y por $(x + 3)$ da de resto 6 y 2, respectivamente.

$$P(x) \text{ polinomio de primer grado} \Rightarrow P(x) = ax + b$$

- ♦ $P(x) : (x - 1)$ tiene resto 6 $\Rightarrow P(1) = 6 \Rightarrow a \cdot 1 + b = 6 \Rightarrow a + b = 6$
 - ♦ $P(x) : (x + 3)$ tiene resto 2 $\Rightarrow P(-3) = 2 \Rightarrow a \cdot (-3) + b = 2 \Rightarrow -3a + b = 2$
- Teorema del resto

Luego

$$\begin{cases} a + b = 6 \\ -3a + b = 2 \end{cases} \xrightarrow{\cdot(-1)} \begin{cases} -a - b = -6 \\ -3a + b = 2 \end{cases}$$

$$-4a = -4 \Rightarrow a = \frac{-4}{-4} \Rightarrow a = 1$$

$$\begin{cases} a + b = 6 \\ a = 1 \end{cases} \Rightarrow a + b = 6 \Rightarrow b = 5$$

Por tanto, $P(x) = x + 5$

Ejercicio 9

¿Qué número m se ha de añadir al polinomio $(x^3 + 2x^2)$ para que sea divisible por $(x + 4)$?

$$\diamond P(x) = x^3 + 2x^2 + m$$

$$\begin{aligned} \diamond P(x) \text{ es divisible por } (x + 4) &\Rightarrow P(x) : (x + 4) \text{ tiene resto } 0 \xRightarrow{\text{Teorema del resto}} P(-4) = 0 \Rightarrow \\ &\Rightarrow (-4)^3 + 2 \cdot (-4)^2 + m = 0 \Rightarrow -64 + 32 + m = 0 \Rightarrow m = 32 \end{aligned}$$

Ejercicio 10

Determina los coeficientes m y n para que el polinomio $P(x) = x^5 + mx^3 + n$ sea divisible por $(x + 1)$ y $(x - 1)$.

$$\begin{aligned} \diamond P(x) \text{ es divisible por } (x + 1) &\Rightarrow P(x) : (x + 1) \text{ tiene resto } 0 \xRightarrow{\text{Teorema del resto}} P(-1) = 0 \Rightarrow \\ &\Rightarrow (-1)^5 + m \cdot (-1)^3 + n = 0 \Rightarrow -1 - m + n = 0 \Rightarrow -m + n = 1 \end{aligned}$$

$$\begin{aligned} \diamond P(x) \text{ es divisible por } (x - 1) &\Rightarrow P(x) : (x - 1) \text{ tiene resto } 0 \xRightarrow{\text{Teorema del resto}} P(1) = 0 \Rightarrow \\ &\Rightarrow 1^5 + m \cdot 1^3 + n = 0 \Rightarrow 1 + m + n = 0 \Rightarrow m + n = -1 \end{aligned}$$

Luego

$$\begin{aligned} \left. \begin{array}{l} -m + n = 1 \\ m + n = -1 \end{array} \right\} \Rightarrow \begin{array}{l} -m + n = 1 \\ n = 0 \end{array} \Rightarrow -m = 1 \Rightarrow m = -1 \\ 2n = 0 \Rightarrow n = 0 \end{aligned}$$

Ejercicio 11

Determina los valores de m y n en el polinomio $P(x) = 3x^3 + mx^2 + nx + 4$ sabiendo que $(x - 1)$ es un factor de $P(x)$ y que si se le divide entre $(x + 2)$ se obtiene de resto 24.

$$\begin{aligned} \diamond (x - 1) \text{ es un factor de } P(x) &\xRightarrow{\text{Teorema del factor}} P(1) = 0 \Rightarrow 3 \cdot 1^3 + m \cdot 1^2 + n \cdot 1 + 4 = 0 \Rightarrow \\ &\Rightarrow 3 + m + n + 4 = 0 \Rightarrow m + n = -7 \end{aligned}$$

$$\begin{aligned} \diamond P(x) : (x + 2) \text{ tiene resto } 24 &\xRightarrow{\text{Teorema del resto}} P(-2) = 24 \Rightarrow 3 \cdot (-2)^3 + m \cdot (-2)^2 + n \cdot (-2) + 4 = 24 \Rightarrow \\ &\Rightarrow -24 + 4m - 2n + 4 = 24 \Rightarrow 4m - 2n = 44 \xRightarrow{(:2)} 2m - n = 22 \end{aligned}$$

Luego

$$\begin{aligned} \left. \begin{array}{l} m + n = -7 \\ 2m - n = 22 \end{array} \right\} \Rightarrow \begin{array}{l} m + n = -7 \\ n = 0 \end{array} \Rightarrow 5 + n = -7 \Rightarrow n = -12 \\ 3m = 15 \Rightarrow m = 5 \end{aligned}$$

Ejercicio 12

Calcula cuánto tienen que valer a y b en el polinomio $P(x) = ax^4 - 10x^3 + x^2 + bx - 2$ para que sea divisible por $(x+1)$ y tenga a 2 como raíz.

- ♦ $P(x)$ es divisible por $(x+1) \Rightarrow P(x):(x+1)$ tiene resto 0 $\xRightarrow{\text{Teorema del resto}} P(-1) = 0 \Rightarrow$
 $\Rightarrow a(-1)^4 - 10 \cdot (-1)^3 + (-1)^2 + b \cdot (-1) - 2 = 0 \Rightarrow a + 10 + 1 - b - 2 = 0 \Rightarrow a - b = -9$
- ♦ 2 es raíz de $P(x) \Rightarrow P(2) = 0 \Rightarrow a \cdot 2^4 - 10 \cdot 2^3 + 2^2 + b \cdot 2 - 2 = 0 \Rightarrow$
 $\Rightarrow 16a - 80 + 4 + 2b - 2 = 0 \Rightarrow 16a + 2b = 78 \xrightarrow{(:2)} 8a + b = 39$

Luego

$$\begin{cases} a - b = -9 \\ 8a + b = 39 \end{cases}$$

$$9a = 30 \Rightarrow a = \frac{30}{9} \Rightarrow a = \frac{10}{3}$$

$$\left. \begin{array}{l} a - b = -9 \\ a = \frac{10}{3} \end{array} \right\} \Rightarrow \frac{10}{3} - b = -9 \Rightarrow \frac{10}{3} + 9 = b \Rightarrow b = \frac{37}{3}$$

Ejercicio 13

Determina m y n en el polinomio $P(x) = x^3 + mx^2 + x + n$ sabiendo que 3 es raíz de $P(x)$ y que al dividirlo entre $(x+2)$ se obtiene de resto 5.

- ♦ 3 es raíz de $P(x) \Rightarrow P(3) = 0 \Rightarrow 3^3 + m \cdot 3^2 + 3 + n = 0 \Rightarrow 27 + 9m + 3 + n = 0 \Rightarrow$
 $\Rightarrow 9m + n = -30$
- ♦ $P(x):(x+2)$ tiene resto 5 $\xRightarrow{\text{Teorema del resto}} P(-2) = 5 \Rightarrow (-2)^3 + m \cdot (-2)^2 + (-2) + n = 5 \Rightarrow$
 $\Rightarrow -8 + 4m - 2 + n = 5 \Rightarrow 4m + n = 15$

$$\begin{cases} 9m + n = -30 \\ 4m + n = 15 \end{cases} \xrightarrow{(-1)} \begin{cases} 9m + n = -30 \\ -4m - n = -15 \end{cases}$$

$$\hline 5m = -45 \Rightarrow m = -9$$

$$\left. \begin{array}{l} 9m + n = -30 \\ m = -9 \end{array} \right\} \Rightarrow -81 + n = -30 \Rightarrow n = 81 - 30 \Rightarrow n = 51$$

Ejercicio 14

Calcula el valor que debemos dar a m en el polinomio $P(x) = x^4 - mx^2 + 3mx - 1$ para que al dividirlo por $(x - 2)$ se obtenga un resto igual al término independiente del polinomio.

$$P(x) : (x - 2) \text{ tiene resto } -1 \Rightarrow P(2) = -1 \Rightarrow 2^4 - m \cdot 2^2 + 3m \cdot 2 - 1 = -1 \Rightarrow$$

Teorema del resto

$$\Rightarrow 16 - 4m + 6m - 1 = -1 \Rightarrow 2m = -16 \Rightarrow m = -8$$

Ejercicio 15

Halla a y b para que al dividir $(x^3 + ax^2 + 2x + b)$ entre $(x^2 + x + 1)$ se obtenga de resto $(-3x + 5)$.

$$\begin{array}{r}
 x^3 \quad + ax^2 \quad + 2x \quad + b \\
 -x^3 \quad - 1x^2 \quad - 1x \quad \\
 \hline
 (a-1)x^2 \quad + 1x \quad + b \\
 - (a-1)x^2 \quad + (1-a)x \quad + (1-a) \\
 \hline
 (2-a)x \quad + (-a+b+1)
 \end{array}
 \left| \begin{array}{l}
 x^2 + x + 1 \\
 \hline
 x + (a-1)
 \end{array} \right.$$

$$\text{Resto} = (-3x + 5) \Leftrightarrow -3x + 5 = (2-a)x + (-a+b+1) \Leftrightarrow \begin{cases} 2-a = -3 \\ -a+b+1 = 5 \end{cases} \Leftrightarrow \begin{cases} a = 5 \\ b = 9 \end{cases}$$