Ejercicio 1

a) $\log_3 27 = y \Rightarrow 3^y = 27 \Rightarrow 3^y = 3^3 \Rightarrow \boxed{y = 3}$

b) $\log_2 128 = y \Rightarrow 2^y = 128 \Rightarrow 2^y = 2^7 \Rightarrow \boxed{y = 7}$

c) $\log_{\frac{1}{2}} 64 = y \Rightarrow \left(\frac{1}{2}\right)^y = 64 \Rightarrow \left(2^{-1}\right)^y = 2^6 \Rightarrow 2^{-y} = 2^6 \Rightarrow -y = 6 \Rightarrow \boxed{y = -6}$

d) $\log_{\sqrt{2}} 32 = y \Rightarrow \left(\sqrt{2}\right)^y = 32 \Rightarrow \left(2^{1/2}\right)^y = 2^5 \Rightarrow 2^{y/2} = 2^5 \Rightarrow \frac{y}{2} = 5 \Rightarrow \boxed{y = 10}$

e) $\log_{\frac{1}{3}} \sqrt[3]{9} = y \Rightarrow \left(\frac{1}{3}\right)^y = \sqrt[3]{9} \Rightarrow 3^{-y} = \sqrt[3]{3^2} \Rightarrow 3^{-y} = 3^{2/3} \Rightarrow -y = \frac{2}{3} \Rightarrow \boxed{y = -\frac{2}{3}}$

f) $\log_{2\sqrt{2}} 0,25 = y \Rightarrow \left(2\sqrt{2}\right)^y = 0,25 \Rightarrow \left(2^1 \cdot 2^{1/2}\right)^y = \frac{1}{4} \Rightarrow (2^{3/2})^y = \frac{1}{2^2} \Rightarrow 2^{\frac{3y}{2}} = 2^{-2} \Rightarrow$

$\Rightarrow \frac{3y}{2} = -2 \Rightarrow \boxed{y = -\frac{4}{3}}$

g) $\log_{\frac{1}{2}} \frac{1}{2\sqrt{8}} = y \Rightarrow \left(\frac{1}{2}\right)^y = \frac{1}{2\sqrt{8}} \Rightarrow (2^{-1})^y = \frac{1}{2 \cdot \sqrt{2^3}} \Rightarrow 2^{-y} = \frac{1}{2^1 \cdot 2^{3/2}} \Rightarrow 2^{-y} = \frac{1}{2^{5/2}} \Rightarrow$

$\Rightarrow 2^{-y} = 2^{-5/2} \Rightarrow \boxed{y = \frac{5}{2}}$

h) $\log_{0,5} \sqrt[3]{16} = y \Rightarrow 0,5^y = \sqrt[3]{16} \Rightarrow \left(\frac{1}{2}\right)^y = \sqrt[3]{2^4} \Rightarrow (2^{-1})^y = 2^{4/3} \Rightarrow 2^{-y} = 2^{4/3} \Rightarrow -y = \frac{4}{3} \Rightarrow$

$\Rightarrow \boxed{y = -\frac{4}{3}}$

i) $\ln \sqrt[5]{e^2} = y \Rightarrow e^y = \sqrt[5]{e^2} \Rightarrow e^y = e^{2/5} \Rightarrow \boxed{y = \frac{2}{5}}$

j) $\ln \frac{e^2}{\sqrt{e}} = y \Rightarrow e^y = \frac{e^2}{\sqrt{e}} \Rightarrow e^y = \frac{e^2}{e^{1/2}} \Rightarrow e^y = e^{3/2} \Rightarrow \boxed{y = \frac{3}{2}}$

k) $\log\ 0,0001 = y \Rightarrow 10^y = 0,0001 \Rightarrow 10^y = 10^{-4} \Rightarrow \boxed{y = -4}$

l) $\log 0 = \nexists$ (no existe)

m) $\log\ (-10)^6 = y \Rightarrow 10^y = (-10)^6 \Rightarrow 10^y = 10^6 \Rightarrow \boxed{y = 6}$

n) $\log\ (-10^6) = \nexists \quad (-10^6 < 0)$

o) $\log_5 5\sqrt{5} = y \Rightarrow 5^y = 5\sqrt{5} \Rightarrow 5^y = 5 \cdot 5^{1/2} \Rightarrow 5^y = 5^{3/2} \Rightarrow \boxed{y = 3/2}$

p) $\log\sqrt{0,01}=y \Rightarrow 10^y=\sqrt{0,01} \Rightarrow 10^y=\sqrt{10^{-2}} \Rightarrow 10^y=10^{-1} \Rightarrow \boxed{y=-1}$

q) $\log_6\sqrt[5]{216^{-1}}=y \Rightarrow 6^y=\sqrt[5]{216^{-1}} \Rightarrow 6^y=\sqrt[5]{(6^3)^{-1}} \Rightarrow 6^y=\sqrt[5]{6^{-3}} \Rightarrow 6^y=6^{-3/5} \Rightarrow \boxed{y=-\frac{3}{5}}$

r) $\log_{\sqrt{\frac{1}{5}}}0,04=y \Rightarrow \left(\sqrt{\frac{1}{5}}\right)^y=0,04 \Rightarrow \left(\sqrt{5^{-1}}\right)^y=\frac{1}{25} \Rightarrow (5^{-1/2})^y=\frac{1}{5^2} \Rightarrow 5^{-y/2}=5^{-2} \Rightarrow$

$\Rightarrow -\frac{y}{2}=-2 \Rightarrow \boxed{y=4}$

s) $\log_4\frac{1}{\sqrt[3]{1024}}=y \Rightarrow 4^y=\frac{1}{\sqrt[3]{1024}} \Rightarrow (2^2)^y=\frac{1}{\sqrt[3]{2^{10}}} \Rightarrow 2^{2y}=2^{-10/3} \Rightarrow 2y=-\frac{10}{3} \Rightarrow \boxed{y=-\frac{5}{3}}$

t) $\log_{128}\sqrt[3]{2}=y \Rightarrow 128^y=\sqrt[3]{2} \Rightarrow (2^7)^y=2^{1/3} \Rightarrow 2^{7y}=2^{1/3} \Rightarrow 7y=\frac{1}{3} \Rightarrow \boxed{y=\frac{1}{21}}$

u) $\log_{\frac{1}{9}}\frac{\sqrt[4]{3}}{9}=y \Rightarrow \left(\frac{1}{9}\right)^y=\frac{\sqrt[4]{3}}{9} \Rightarrow (3^{-2})^y=\frac{3^{1/4}}{3^2} \Rightarrow 3^{-2y}=3^{-7/4} \Rightarrow -2y=-\frac{7}{4} \Rightarrow \boxed{y=\frac{7}{8}}$

v) $\log_3\frac{\sqrt[4]{3}}{\sqrt{27}}=y \Rightarrow 3^y=\frac{\sqrt[4]{3}}{\sqrt{27}} \Rightarrow 3^y=\frac{\sqrt[4]{3}}{\sqrt{3^3}} \Rightarrow 3^y=\frac{3^{1/4}}{3^{3/2}} \Rightarrow 3^y=3^{-5/4} \Rightarrow \boxed{y=-\frac{5}{4}}$

w) $\log_2(-16)=\nexists$

x) $\ln\frac{1}{e^3}=y \Rightarrow e^y=\frac{1}{e^3} \Rightarrow e^y=e^{-3} \Rightarrow \boxed{y=-3}$

y) $\log_{-3}81=\nexists$