

Ejercicio 2

$$a) \log_{25} \frac{1}{\sqrt[5]{5}} - \log_3 \sqrt{243} + \log_{16} \frac{1}{4} = -\frac{1}{10} - \frac{5}{2} + \left(-\frac{1}{2}\right) = \frac{-1-25-5}{10} = \boxed{-\frac{31}{10}}$$

$$\cdot \log_{25} \frac{1}{\sqrt[5]{5}} = y \Rightarrow 25^y = \frac{1}{\sqrt[5]{5}} \Rightarrow (5^2)^y = \frac{1}{5^{1/5}} \Rightarrow 5^{2y} = 5^{-1/5} \Rightarrow 2y = -\frac{1}{5} \Rightarrow y = -\frac{1}{10}$$

$$\cdot \log_3 \sqrt{243} = y \Rightarrow 3^y = \sqrt{243} \Rightarrow 3^y = \sqrt{3^5} \Rightarrow 3^y = 3^{5/2} \Rightarrow y = \frac{5}{2}$$

$$\cdot \log_{16} \frac{1}{4} = y \Rightarrow 16^y = \frac{1}{4} \Rightarrow (4^y)^2 = 4^{-1} \Rightarrow 4^{2y} = 4^{-1} \Rightarrow 2y = -1 \Rightarrow y = -\frac{1}{2}$$

$$b) \log_2 \sqrt[6]{0,5} - \log_{49} \frac{1}{7} - \log_{216} 6 - \log_4 \sqrt{2\sqrt{2}} = -\frac{1}{6} - \left(-\frac{1}{2}\right) - \frac{1}{3} - \frac{3}{8} = \frac{-4+12-8-9}{24} = -\frac{9}{24} = \boxed{-\frac{3}{8}}$$

$$\cdot \log_2 \sqrt[6]{0,5} = y \Rightarrow 2^y = \sqrt[6]{0,5} \Rightarrow 2^y = \sqrt[6]{\frac{1}{2}} \Rightarrow 2^y = \sqrt[6]{2^{-1}} \Rightarrow 2^y = 2^{-1/6} \Rightarrow y = -\frac{1}{6}$$

$$\cdot \log_{49} \frac{1}{7} = y \Rightarrow 49^y = \frac{1}{7} \Rightarrow (7^2)^y = 7^{-1} \Rightarrow 7^{2y} = 7^{-1} \Rightarrow 2y = -1 \Rightarrow y = -\frac{1}{2}$$

$$\cdot \log_{216} 6 = y \Rightarrow 216^y = 6 \Rightarrow 6^{3y} = 6^1 \Rightarrow 3y = 1 \Rightarrow y = \frac{1}{3}$$

$$\cdot \log_4 \sqrt{2\sqrt{2}} = y \Rightarrow 4^y = \sqrt{2\sqrt{2}} \Rightarrow (2^2)^y = \sqrt{\sqrt{2^2} \cdot 2} \Rightarrow 2^{2y} = \sqrt[4]{2^3} \Rightarrow 2^{2y} = 2^{3/4} \Rightarrow$$

$$\Rightarrow 2y = \frac{3}{4} \Rightarrow y = \frac{3}{8}$$

Ejercicio 3

$$a) \log_x 7 = -2 \Rightarrow x^{-2} = 7 \Rightarrow \frac{1}{x^2} = 7 \Rightarrow x^2 = \frac{1}{7} \underset{(x > 0)}{\Rightarrow} x = +\sqrt{\frac{1}{7}} \Rightarrow x = \frac{1}{\sqrt{7}} \Rightarrow$$

$$\underset{\text{racionalizar}}{\Rightarrow} x = \frac{1 \cdot \sqrt{7}}{\sqrt{7} \cdot \sqrt{7}} = \frac{\sqrt{7}}{7} \Rightarrow \boxed{x = \frac{\sqrt{7}}{7}}$$

$$b) \log_x 7 = \frac{1}{2} \Rightarrow x^{1/2} = 7 \Rightarrow \sqrt{x} = 7 \Rightarrow (\sqrt{x})^2 = 7^2 \Rightarrow \boxed{x = 49}$$

$$c) \log_7 x^4 = 2 \Rightarrow 7^2 = x^4 \Rightarrow x = \pm \sqrt[4]{7^2} \underset{\text{simplif.}}{\Rightarrow} \boxed{x = \pm \sqrt{7}}$$

$$d) \log_x \frac{1}{2} = -\frac{1}{4} \Rightarrow x^{-1/4} = \frac{1}{2} \Rightarrow \frac{1}{x^{1/4}} = \frac{1}{2} \Rightarrow x^{1/4} = 2 \Rightarrow \sqrt[4]{x} = 2 \Rightarrow (\sqrt[4]{x})^4 = 2^4 \Rightarrow \boxed{x = 16}$$

$$e) \log_7(7x) = 2 \Rightarrow 7^2 = 7x \Rightarrow 49 = 7x \Rightarrow \boxed{x = 7}$$

$$f) \log_x \frac{1}{3} = -\frac{1}{2} \Rightarrow x^{-1/2} = \frac{1}{3} \Rightarrow \frac{1}{x^{1/2}} = \frac{1}{3} \Rightarrow x^{1/2} = 3 \Rightarrow \sqrt{x} = 3 \Rightarrow (\sqrt{x})^2 = 3^2 \Rightarrow \boxed{x = 9}$$