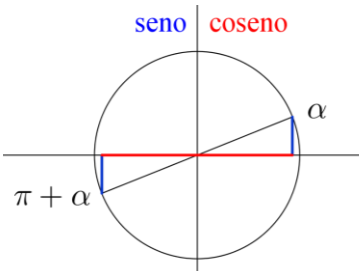
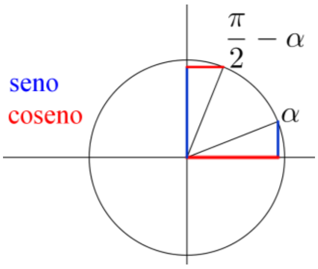
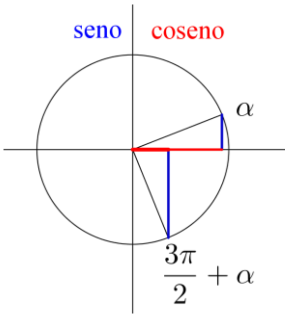
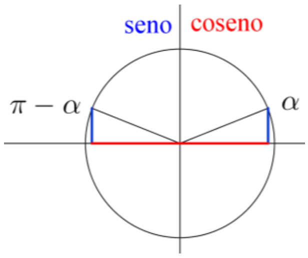
Ejercicio 46

a) $\dfrac{\cos(\pi+\alpha)-\operatorname{sen}(\pi/2-\alpha)}{\operatorname{sen}(3\pi/2+\alpha)+\cos(\pi-\alpha)}=\dfrac{-\cos\alpha-\cos\alpha}{-\cos\alpha-\cos\alpha}=\dfrac{-2\cos\alpha}{-2\cos\alpha}=1$









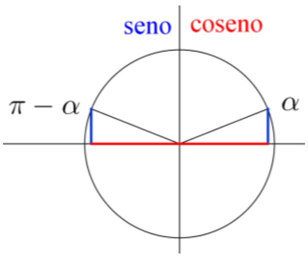
b) $(2-\operatorname{cosec}^2\alpha):\dfrac{(\operatorname{sen}^4\alpha-\cos^4\alpha)}{\operatorname{sen}^2\alpha}=$

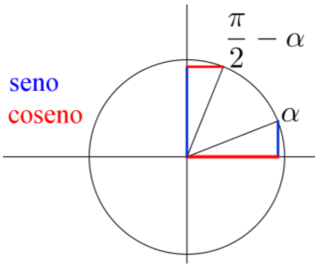
$$=\left(\frac{2}{1}-\frac{1}{\operatorname{sen}^2\alpha}\right):\frac{\overbrace{(\operatorname{sen}^2\alpha+\cos^2\alpha)}^{1}\cdot(\operatorname{sen}^2\alpha-\cos^2\alpha)}{\operatorname{sen}^2\alpha}=\frac{2\operatorname{sen}^2\alpha-1}{\operatorname{sen}^2\alpha}:\frac{\operatorname{sen}^2\alpha-\cos^2\alpha}{\operatorname{sen}^2\alpha}=$$

$$=\frac{\operatorname{sen}^2\alpha\cdot(2\operatorname{sen}^2\alpha-1)}{\operatorname{sen}^2\alpha\cdot(\operatorname{sen}^2\alpha-\cos^2\alpha)}=\frac{2\operatorname{sen}^2\alpha-1}{\operatorname{sen}^2\alpha-\underbrace{\cos^2\alpha}_{1-\operatorname{sen}^2\alpha}}=\frac{2\operatorname{sen}^2\alpha-1}{\operatorname{sen}^2\alpha-(1-\operatorname{sen}^2\alpha)}=\frac{2\operatorname{sen}^2\alpha-1}{\operatorname{sen}^2\alpha-1+\operatorname{sen}^2\alpha}=\frac{2\operatorname{sen}^2\alpha-1}{2\operatorname{sen}^2\alpha-1}=1$$

c) $\dfrac{\cos^3\alpha+\cos\alpha\cdot\operatorname{sen}^2\alpha}{\operatorname{sen}^3\alpha+\cos^2\alpha\cdot\operatorname{sen}\alpha}\underset{\text{factor común}}{=}\dfrac{\cos\alpha\cdot(\cos^2\alpha+\operatorname{sen}^2\alpha)}{\operatorname{sen}\alpha\cdot(\operatorname{sen}^2\alpha+\cos^2\alpha)}=\dfrac{\cos\alpha\cdot 1}{\operatorname{sen}\alpha\cdot 1}=\dfrac{\cos\alpha}{\operatorname{sen}\alpha}=\operatorname{cotg}\alpha$

d) $\dfrac{\operatorname{sen}^2(\pi-\alpha)\cdot\cos\left(\dfrac{\pi}{2}-\alpha\right)}{\operatorname{sen}\alpha\cdot(1-\cos^2\alpha)}=\dfrac{\operatorname{sen}^2\alpha\cdot\operatorname{sen}\alpha}{\operatorname{sen}\alpha\cdot\operatorname{sen}^2\alpha}=\dfrac{\operatorname{sen}^3\alpha}{\operatorname{sen}^3\alpha}=1$



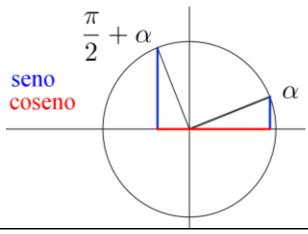


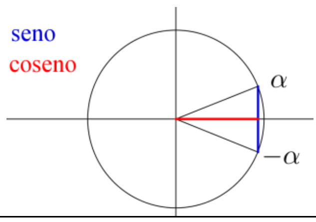
e) $\dfrac{\operatorname{tg}\left(\dfrac{\pi}{2}+\alpha\right)\cdot\operatorname{tg}(-\alpha)}{\underbrace{\operatorname{cotg}(\pi-\alpha)\cdot\operatorname{tg}(\pi-\alpha)}_{1}}=\dfrac{-\operatorname{cotg}\alpha\cdot(-\operatorname{tg}\alpha)}{1}=\operatorname{cotg}\alpha\cdot\operatorname{tg}\alpha=1$

La tangente y la cotangente son R.T. inversas, por tanto, $\operatorname{cotg} A\cdot\operatorname{tg} A=1$ para cualquier ángulo A

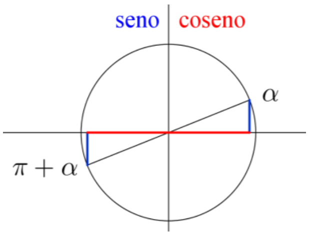
$\operatorname{tg}\left(\dfrac{\pi}{2}+\alpha\right)=\dfrac{\operatorname{sen}(\pi/2+\alpha)}{\cos(\pi/2+\alpha)}=\dfrac{\cos\alpha}{-\operatorname{sen}\alpha}=-\operatorname{cotg}\alpha$

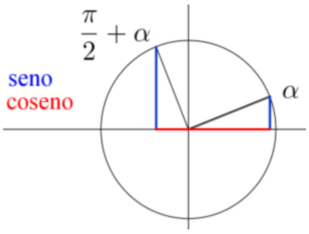
$\operatorname{tg}(-\alpha)=\dfrac{\operatorname{sen}(-\alpha)}{\cos(-\alpha)}=\dfrac{-\operatorname{sen}\alpha}{\cos\alpha}=-\operatorname{tg}\alpha$

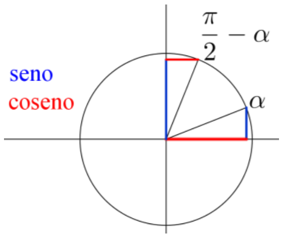




f) $$\frac{\operatorname{sen}(\pi+\alpha)\cdot\cos\left(\frac{\pi}{2}+\alpha\right)}{\underbrace{1-\cos^2\alpha}_{\operatorname{sen}^2\alpha}}-\cos^2\left(\frac{\pi}{2}-\alpha\right)=\frac{-\operatorname{sen}\alpha\cdot(-\operatorname{sen}\alpha)}{\operatorname{sen}^2\alpha}-\operatorname{sen}^2\alpha=\frac{\operatorname{sen}^2\alpha}{\operatorname{sen}^2\alpha}-\operatorname{sen}^2\alpha=1-\operatorname{sen}^2\alpha=\cos^2\alpha$$





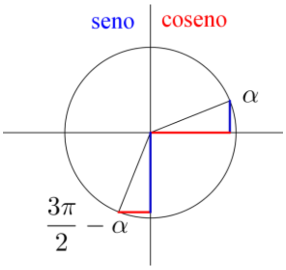


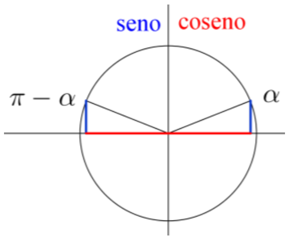
g) $$\frac{\operatorname{tg}\left(-\frac{\pi}{2}-\alpha\right)\cdot\operatorname{tg}(\pi-\alpha)}{\operatorname{sen}\left(\frac{\pi}{2}-\alpha\right)}\cdot\cos\alpha\underset{\uparrow\atop -\frac{\pi}{2}=\frac{3\pi}{2}}{=}\frac{\operatorname{tg}\left(\frac{3\pi}{2}-\alpha\right)\cdot\operatorname{tg}(\pi-\alpha)}{\operatorname{sen}\left(\frac{\pi}{2}-\alpha\right)}\cdot\cos\alpha=\frac{\operatorname{cotg}\alpha\cdot(-\operatorname{tg}\alpha)}{\cancel{\cos\alpha}}\cdot\cancel{\cos\alpha}=$$

$$=-\operatorname{cotg}\alpha\cdot\operatorname{tg}\alpha=-1$$

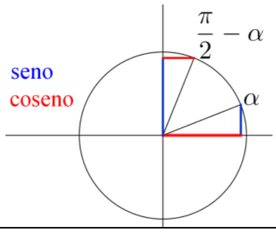
$$\operatorname{tg}\left(\frac{3\pi}{2}-\alpha\right)=\frac{\operatorname{sen}(3\pi/2-\alpha)}{\cos(3\pi/2-\alpha)}=\frac{-\cos\alpha}{-\operatorname{sen}\alpha}=\operatorname{cotg}\alpha$$

$$\operatorname{tg}(\pi-\alpha)=\frac{\operatorname{sen}(\pi-\alpha)}{\cos(\pi-\alpha)}=\frac{\operatorname{sen}\alpha}{-\cos\alpha}=-\operatorname{tg}\alpha$$

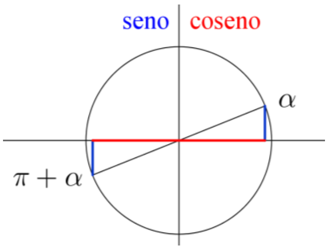


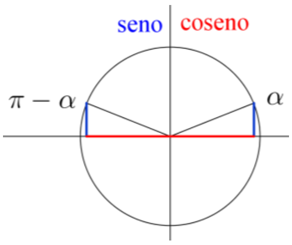


$$\operatorname{sen}\left(\frac{\pi}{2}-\alpha\right)=\cos\alpha$$



h) $$\frac{\operatorname{sen}(\pi+\alpha)\cdot\cos(\pi-\alpha)}{\operatorname{sen}(\pi-\alpha)\cdot\cos(\pi+\alpha)}=\frac{-\operatorname{sen}\alpha\cdot(-\cos\alpha)}{\operatorname{sen}\alpha\cdot(-\cos\alpha)}=\frac{\operatorname{sen}\alpha\cdot\cos\alpha}{-\operatorname{sen}\alpha\cdot\cos\alpha}=-1$$





Ejercicio 47

a) $\operatorname{cosec}^2\alpha - \operatorname{cotg}^2\alpha = 1$

$$\frac{1}{\operatorname{sen}^2\alpha} - \frac{\cos^2\alpha}{\operatorname{sen}^2\alpha} = \frac{1-\cos^2\alpha}{\operatorname{sen}^2\alpha} = \frac{\operatorname{sen}^2\alpha}{\operatorname{sen}^2\alpha} = 1$$

b) $\sec^2\alpha + \operatorname{cosec}^2\alpha = \sec^2\alpha \cdot \operatorname{cosec}^2\alpha$

$$\sec^2\alpha + \operatorname{cosec}^2\alpha = \frac{1}{\cos^2\alpha} + \frac{1}{\operatorname{sen}^2\alpha} = \frac{\operatorname{sen}^2\alpha + \cos^2\alpha}{\cos^2\alpha \cdot \operatorname{sen}^2\alpha} = \frac{1}{\cos^2\alpha \cdot \operatorname{sen}^2\alpha} = \frac{1}{\cos^2\alpha} \cdot \frac{1}{\operatorname{sen}^2\alpha} =$$

$$= \sec^2\alpha \cdot \operatorname{cosec}^2\alpha$$

c) $\operatorname{tg}\alpha + \operatorname{cotg}\alpha = \sec\alpha \cdot \operatorname{cosec}\alpha$

$$\frac{\operatorname{sen}\alpha}{\cos\alpha} + \frac{\cos\alpha}{\operatorname{sen}\alpha} = \frac{\operatorname{sen}^2\alpha + \operatorname{sen}^2\alpha}{\cos\alpha \cdot \operatorname{sen}\alpha} = \frac{1}{\cos\alpha \cdot \operatorname{sen}\alpha} = \frac{1}{\cos\alpha} \cdot \frac{1}{\operatorname{sen}\alpha} = \sec\alpha \cdot \operatorname{cosec}\alpha$$

d) $\operatorname{cotg}^2\alpha = \cos^2\alpha + (\operatorname{cotg}\alpha \cdot \cos\alpha)^2$

$$\cos^2\alpha + (\operatorname{cotg}\alpha \cdot \cos\alpha)^2 = \cos^2\alpha + \operatorname{cotg}^2\alpha \cdot \cos^2\alpha = \cos^2\alpha \cdot (1 + \operatorname{cotg}^2\alpha) = \cos^2\alpha \cdot \operatorname{cosec}^2\alpha =$$

$$= \cos^2\alpha \cdot \frac{1}{\operatorname{sen}^2\alpha} = \frac{\cos^2\alpha}{\operatorname{sen}^2\alpha} = \operatorname{cotg}^2\alpha$$

e) $\operatorname{sen}^2\alpha - \cos^2\alpha = \operatorname{sen}^4\alpha - \cos^4\alpha$

$$\operatorname{sen}^4\alpha - \cos^4\alpha = (\operatorname{sen}^2\alpha - \cos^2\alpha) \cdot \overbrace{(\operatorname{sen}^2\alpha + \cos^2\alpha)}^{1} = \operatorname{sen}^2\alpha - \cos^2\alpha$$

f) $\dfrac{\cos^2\alpha}{1+\operatorname{sen}\alpha} = 1 - \operatorname{sen}\alpha$

$$\frac{\cos^2\alpha}{1+\operatorname{sen}\alpha} = \frac{1-\operatorname{sen}^2\alpha}{1+\operatorname{sen}\alpha} = \frac{(1-\operatorname{sen}\alpha)\cdot(1+\operatorname{sen}\alpha)}{1+\operatorname{sen}\alpha} = 1 - \operatorname{sen}\alpha$$

g) $\dfrac{1+\operatorname{tg}\alpha}{\sec\alpha} = \operatorname{sen}\alpha + \cos\alpha$

$$\frac{1+\dfrac{\operatorname{sen}\alpha}{\cos\alpha}}{\dfrac{1}{\cos\alpha}} = \frac{\dfrac{\cos\alpha + \operatorname{sen}\alpha}{\cos\alpha}}{\dfrac{1}{\cos\alpha}} = \frac{\cos\alpha \cdot (\cos\alpha + \operatorname{sen}\alpha)}{\cos\alpha} = \cos\alpha + \operatorname{sen}\alpha = \operatorname{sen}\alpha + \cos\alpha$$

h) $\dfrac{1}{\sec^2\alpha} = \operatorname{sen}^2\alpha \cdot \cos^2\alpha + \cos^4\alpha$

$$\operatorname{sen}^2\alpha \cdot \cos^2\alpha + \cos^4\alpha \underset{\text{factor común}}{=} \cos^2\alpha \cdot \overbrace{(\operatorname{sen}^2\alpha + \cos^2\alpha)}^{1} = \cos^2\alpha = \frac{1}{\sec^2\alpha}$$

i) $\dfrac{1+\sec\alpha}{1-\sec\alpha}=\dfrac{\cos\alpha+1}{\cos\alpha-1}$

$$\frac{1+\sec\alpha}{1-\sec\alpha}=\frac{1+\dfrac{1}{\cos\alpha}}{1-\dfrac{1}{\cos\alpha}}=\frac{\dfrac{\cos\alpha+1}{\cos\alpha}}{\dfrac{\cos\alpha-1}{\cos\alpha}}=\frac{\cos\alpha\cdot(\cos\alpha+1)}{\cos\alpha\cdot(\cos\alpha-1)}=\frac{\cos\alpha+1}{\cos\alpha-1}$$

j) $2\operatorname{sen}^2\alpha-1=\operatorname{sen}^4\alpha-\cos^4\alpha$

$$\operatorname{sen}^4\alpha-\cos^4\alpha=(\operatorname{sen}^2\alpha-\cos^2\alpha)\cdot\overbrace{(\operatorname{sen}^2\alpha+\cos^2\alpha)}^{1}=\operatorname{sen}^2\alpha-\cos^2\alpha=\operatorname{sen}^2\alpha-(1-\operatorname{sen}^2\alpha)=$$

$$=\operatorname{sen}^2\alpha-1+\operatorname{sen}^2\alpha=2\operatorname{sen}^2\alpha-1$$

k) $\dfrac{\operatorname{sen}\alpha\cdot\cos\alpha}{1-\cos\alpha}-\operatorname{cotg}\alpha=\operatorname{cosec}\alpha-\operatorname{sen}\alpha$

$$\frac{\operatorname{sen}\alpha\cdot\cos\alpha}{1-\cos\alpha}-\operatorname{cotg}\alpha=\frac{\operatorname{sen}\alpha\cdot\cos\alpha}{1-\cos\alpha}-\frac{\cos\alpha}{\operatorname{sen}\alpha}=\frac{\overbrace{\operatorname{sen}^2\alpha}^{1-\cos^2\alpha}\cdot\cos\alpha-\cos\alpha+\cos^2\alpha}{(1-\cos\alpha)\cdot\operatorname{sen}\alpha}=$$

$$=\frac{(1-\cos^2\alpha)\cdot\cos\alpha-\cos\alpha+\cos^2\alpha}{(1-\cos\alpha)\cdot\operatorname{sen}\alpha}=\frac{\cos\alpha-\cos^3\alpha-\cos\alpha+\cos^2\alpha}{(1-\cos\alpha)\cdot\operatorname{sen}\alpha}=$$

$$=\frac{\cos^2\alpha-\cos^3\alpha}{(1-\cos\alpha)\cdot\operatorname{sen}\alpha}\underset{\text{factor común}}{=}\frac{\cos^2\alpha\cdot(1-\cos\alpha)}{(1-\cos\alpha)\cdot\operatorname{sen}\alpha}=\frac{\cos^2\alpha}{\operatorname{sen}\alpha}=\frac{1-\operatorname{sen}^2\alpha}{\operatorname{sen}\alpha}=\frac{1}{\operatorname{sen}\alpha}-\frac{\operatorname{sen}^2\alpha}{\operatorname{sen}\alpha}=\operatorname{cosec}\alpha-\operatorname{sen}\alpha$$

k) $\dfrac{\operatorname{sen}\alpha\cdot\cos\alpha}{1-\cos\alpha}-\operatorname{cotg}\alpha=\operatorname{cosec}\alpha-\operatorname{sen}\alpha$ **Otra forma**

$$=\frac{\operatorname{sen}\alpha\cdot\cos\alpha}{1-\cos\alpha}-\operatorname{cotg}\alpha=\frac{\operatorname{sen}\alpha\cdot\cos\alpha}{1-\cos\alpha}-\frac{\cos\alpha}{\operatorname{sen}\alpha}=\frac{\operatorname{sen}\alpha\cdot\cos\alpha\cdot(1+\cos\alpha)}{(1-\cos\alpha)\cdot(1+\cos\alpha)}-\frac{\cos\alpha}{\operatorname{sen}\alpha}=$$

$$=\frac{\operatorname{sen}\alpha\cdot\cos\alpha\cdot(1+\cos\alpha)}{1-\cos^2\alpha}-\frac{\cos\alpha}{\operatorname{sen}\alpha}=\frac{\operatorname{sen}\alpha\cdot\cos\alpha\cdot(1+\cos\alpha)}{\operatorname{sen}^2\alpha}-\frac{\cos\alpha}{\operatorname{sen}\alpha}=\frac{\cos\alpha+\cos^2\alpha}{\operatorname{sen}\alpha}-\frac{\cos\alpha}{\operatorname{sen}\alpha}=$$

$$=\frac{\cos\alpha+\cos^2\alpha-\cos\alpha}{\operatorname{sen}\alpha}=\frac{\cos^2\alpha}{\operatorname{sen}\alpha}=\frac{1-\operatorname{sen}^2\alpha}{\operatorname{sen}\alpha}=\frac{1}{\operatorname{sen}\alpha}-\frac{\operatorname{sen}^2\alpha}{\operatorname{sen}\alpha}=\operatorname{cosec}\alpha-\operatorname{sen}\alpha$$

l) $\dfrac{\sec^2\alpha}{\operatorname{cotg}\alpha}\cdot(1-\operatorname{sen}^2\alpha)\cdot\operatorname{cosec}^2\alpha=\dfrac{\operatorname{cosec}\alpha}{\cos\alpha}$

$$\frac{\sec^2\alpha}{\operatorname{cotg}\alpha}\cdot(1-\operatorname{sen}^2\alpha)\cdot\operatorname{cosec}^2\alpha=\frac{\dfrac{1}{\cos^2\alpha}}{\dfrac{\cos\alpha}{\operatorname{sen}\alpha}}\cdot\cos^2\alpha\cdot\frac{1}{\operatorname{sen}^2\alpha}=\frac{\operatorname{sen}\alpha}{\cos^3\alpha}\cdot\cos^2\alpha\cdot\frac{1}{\operatorname{sen}^2\alpha}=$$

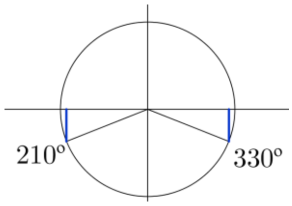
$$=\frac{\operatorname{sen}\alpha\cdot\cos^2\alpha}{\cos^3\alpha\cdot\operatorname{sen}^2\alpha}=\frac{1}{\cos\alpha\cdot\operatorname{sen}\alpha}=\frac{1}{\cos\alpha}\cdot\frac{1}{\operatorname{sen}\alpha}=\frac{1}{\cos\alpha}\cdot\operatorname{cosec}\alpha=\frac{\operatorname{cosec}\alpha}{\cos\alpha}$$

m) $\dfrac{\text{sen}\alpha+\cos\alpha\cdot\text{tg}\alpha}{\cos\alpha}=2\,\text{tg}\alpha$ $\dfrac{\text{sen}\alpha+\cos\alpha\cdot\text{tg}\alpha}{\cos\alpha}=\dfrac{\text{sen}\alpha+\cos\alpha\cdot\dfrac{\text{sen}\alpha}{\cos\alpha}}{\cos\alpha}=\dfrac{\text{sen}\alpha+\text{sen}\alpha}{\cos\alpha}=\dfrac{2\cdot\text{sen}\alpha}{\cos\alpha}=2\,\text{tg}\alpha$
n) $(1-\text{sen}^2\alpha)\cdot\dfrac{1}{\cos\alpha}\cdot\dfrac{1+\cos^2\alpha}{2-\text{sen}^2\alpha}\cdot\text{tg}\,\alpha=\text{sen}\,\alpha$ $(1-\text{sen}^2\alpha)\cdot\dfrac{1}{\cos\alpha}\cdot\dfrac{1+\cos^2\alpha}{2-\text{sen}^2\alpha}\cdot\text{tg}\,\alpha=\cos^2\alpha\cdot\dfrac{1}{\cos\alpha}\cdot\dfrac{1+\cos^2\alpha}{2-(1-\cos^2\alpha)}\cdot\dfrac{\text{sen}\,\alpha}{\cos\alpha}=\dfrac{\cos^2\alpha\cdot(1+\cos^2\alpha)\cdot\text{sen}\,\alpha}{\cos^2\alpha\cdot(1+\cos^2\alpha)}=$ $=\text{sen}\,\alpha$
o) $\text{cotg}^2\alpha\cdot\cos^2\alpha-\text{cotg}^2\alpha=-\cos^2\alpha$ $\text{cotg}^2\alpha\cdot\cos^2\alpha-\text{cotg}^2\alpha=\text{cotg}^2\alpha\cdot(\cos^2\alpha-1)=\dfrac{\cos^2\alpha}{\text{sen}^2\alpha}\cdot(-\text{sen}^2\alpha)=-\cos^2\alpha$
p) $\dfrac{\cos^4\alpha-\text{sen}^4\alpha}{\text{sen}\,\alpha\cdot\cos\alpha}=\dfrac{1-\text{tg}^2\alpha}{\text{tg}\,\alpha}$ $\dfrac{\cos^4\alpha-\text{sen}^4\alpha}{\text{sen}\,\alpha\cdot\cos\alpha}=\dfrac{(\cos^2\alpha-\text{sen}^2\alpha)\cdot\overbrace{(\cos^2\alpha+\text{sen}^2\alpha)}^{1}}{\text{sen}\,\alpha\cdot\cos\alpha}=\dfrac{\cos^2\alpha-\text{sen}^2\alpha}{\text{sen}\,\alpha\cdot\cos\alpha}$ $\dfrac{1-\text{tg}^2\alpha}{\text{tg}\,\alpha}=\dfrac{1-\dfrac{\text{sen}^2\alpha}{\cos^2\alpha}}{\dfrac{\text{sen}\,\alpha}{\cos\alpha}}=\dfrac{\dfrac{\cos^2\alpha-\text{sen}^2\alpha}{\cos^2\alpha}}{\dfrac{\text{sen}\,\alpha}{\cos\alpha}}=\dfrac{\cos\alpha\cdot(\cos^2\alpha-\text{sen}^2\alpha)}{\text{sen}\,\alpha\cdot\cos^2\alpha}=\dfrac{\cos^2\alpha-\text{sen}^2\alpha}{\text{sen}\,\alpha\cdot\cos\alpha}$
p) $\dfrac{\cos^4\alpha-\text{sen}^4\alpha}{\text{sen}\,\alpha\cdot\cos\alpha}=\dfrac{1-\text{tg}^2\alpha}{\text{tg}\,\alpha}$ **Otra forma** $\dfrac{\cos^4\alpha-\text{sen}^4\alpha}{\text{sen}\,\alpha\cdot\cos\alpha}=\dfrac{(\cos^2\alpha-\text{sen}^2\alpha)\cdot\overbrace{(\cos^2\alpha+\text{sen}^2\alpha)}^{1}}{\text{sen}\,\alpha\cdot\cos\alpha}=\dfrac{\cos^2\alpha-\text{sen}^2\alpha}{\text{sen}\,\alpha\cdot\cos\alpha}=\dfrac{\dfrac{\cos^2\alpha-\text{sen}^2\alpha}{\cos^2\alpha}}{\dfrac{\text{sen}\,\alpha\cdot\cos\alpha}{\cos^2\alpha}}=$ $=\dfrac{\dfrac{\cos^2\alpha}{\cos^2\alpha}-\dfrac{\text{sen}^2\alpha}{\cos^2\alpha}}{\dfrac{\text{sen}\,\alpha}{\cos\alpha}}=\dfrac{1-\text{tg}^2\alpha}{\text{tg}\,\alpha}$
q) $(1+\text{tg}\,\alpha)\cdot(1+\text{cotg}\,\alpha)=\dfrac{(\text{sen}\,\alpha+\cos\alpha)^2}{\text{sen}\,\alpha\cdot\cos\alpha}$ $(1+\text{tg}\,\alpha)\cdot(1+\text{cotg}\,\alpha)=\left(1+\dfrac{\text{sen}\,\alpha}{\cos\alpha}\right)\cdot\left(1+\dfrac{\cos\alpha}{\text{sen}\,\alpha}\right)=\left(\dfrac{\cos\alpha+\text{sen}\,\alpha}{\cos\alpha}\right)\cdot\left(\dfrac{\text{sen}\,\alpha+\cos\alpha}{\text{sen}\,\alpha}\right)=$ $=\left(\dfrac{\text{sen}\,\alpha+\cos\alpha}{\cos\alpha}\right)\cdot\left(\dfrac{\text{sen}\,\alpha+\cos\alpha}{\text{sen}\,\alpha}\right)=\dfrac{(\text{sen}\,\alpha+\cos\alpha)\cdot(\text{sen}\,\alpha+\cos\alpha)}{\cos\alpha\cdot\text{sen}\,\alpha}=\dfrac{(\text{sen}\,\alpha+\cos\alpha)^2}{\text{sen}\,\alpha\cdot\cos\alpha}$

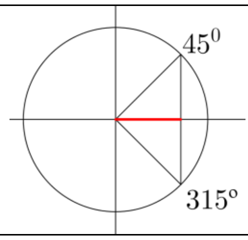
r)	$\dfrac{\text{sen}\,\alpha+\text{cotg}\,\alpha}{\text{tg}\,\alpha+\text{cosec}\,\alpha}=\text{sen}\,\alpha\cdot\text{cotg}\,\alpha$ ▪ $\dfrac{\text{sen}\,\alpha+\text{cotg}\,\alpha}{\text{tg}\,\alpha+\text{cosec}\,\alpha}=\dfrac{\text{sen}\,\alpha+\dfrac{\cos\alpha}{\text{sen}\,\alpha}}{\dfrac{\text{sen}\,\alpha}{\cos\alpha}+\dfrac{1}{\text{sen}\,\alpha}}=\dfrac{\dfrac{\text{sen}^2\alpha+\cos\alpha}{\text{sen}\,\alpha}}{\dfrac{\text{sen}^2\alpha+\cos\alpha}{\cos\alpha\cdot\text{sen}\,\alpha}}=\dfrac{\cos\alpha\cdot\cancel{\text{sen}\,\alpha}\cdot\cancel{(\text{sen}^2\alpha+\cos\alpha)}}{\cancel{\text{sen}\,\alpha}\cdot\cancel{(\text{sen}^2\alpha+\cos\alpha)}}=\cos\alpha$ ▪ $\text{sen}\,\alpha\cdot\text{cotg}\,\alpha=\text{sen}\,\alpha\cdot\dfrac{\cos\alpha}{\text{sen}\,\alpha}=\cos\alpha$
s)	$\dfrac{\sec\alpha-\cos\alpha}{\text{cosec}\,\alpha-\text{sen}\,\alpha}=\text{tg}^3\alpha$ $\dfrac{\sec\alpha-\cos\alpha}{\text{cosec}\,\alpha-\text{sen}\,\alpha}=\dfrac{\dfrac{1}{\cos\alpha}-\cos\alpha}{\dfrac{1}{\text{sen}\,\alpha}-\text{sen}\,\alpha}=\dfrac{\dfrac{1-\cos^2\alpha}{\cos\alpha}}{\dfrac{1-\text{sen}^2\alpha}{\text{sen}\,\alpha}}=\dfrac{\dfrac{\text{sen}^2\alpha}{\cos\alpha}}{\dfrac{\cos^2\alpha}{\text{sen}\,\alpha}}=\dfrac{\text{sen}^3\alpha}{\cos^3\alpha}=\text{tg}^3\alpha$
t)	$\dfrac{\text{sen}^2\alpha+\text{sen}\,\alpha\cdot\cos\alpha+\cos^2\alpha}{\cos^2\alpha}=1+\text{tg}\,\alpha+\text{tg}^2\alpha$ $1+\text{tg}\,\alpha+\text{tg}^2\alpha=1+\dfrac{\text{sen}\,\alpha}{\cos\alpha}+\dfrac{\text{sen}^2\alpha}{\cos^2\alpha}=\dfrac{\cos^2\alpha+\text{sen}\,\alpha\cdot\cos\alpha+\text{sen}^2\alpha}{\cos^2\alpha}=\dfrac{\text{sen}^2\alpha+\text{sen}\,\alpha\cdot\cos\alpha+\cos^2\alpha}{\cos^2\alpha}$

Ejercicio 55

1) $\operatorname{sen} x = -\dfrac{1}{2} \Rightarrow x = \operatorname{arcsen}\left(-\dfrac{1}{2}\right) \Rightarrow x = \begin{cases} 210^{\circ} + 360^{\circ} k \\ 330^{\circ} + 360^{\circ} k \end{cases} \quad k \in \mathbb{Z}$



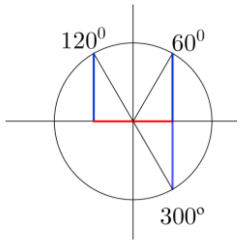
2) $\cos x = \dfrac{\sqrt{2}}{2} \Rightarrow x = \arccos\left(\dfrac{\sqrt{2}}{2}\right) \Rightarrow x = \begin{cases} 45^{\circ} + 360^{\circ} k \\ 315^{\circ} + 360^{\circ} k \end{cases} \quad k \in \mathbb{Z}$



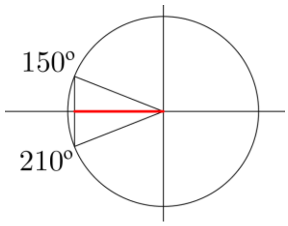
3) $\sec x = -\dfrac{1}{2} \Rightarrow \cos x = -2 \Rightarrow$ no tiene solución ya que $-1 \le \cos x \le 1 \quad \forall x$

4) $\operatorname{tg} x = -\sqrt{3} \Rightarrow x = \operatorname{arctg}\left(-\sqrt{3}\right) \Rightarrow x = \begin{cases} 300^{\circ} + 360^{\circ} k \\ 120^{\circ} + 360^{\circ} k \end{cases} \quad k \in \mathbb{Z} \Rightarrow$

$\Rightarrow x = 120^{\circ} + 180^{\circ} k \quad k \in \mathbb{Z}$

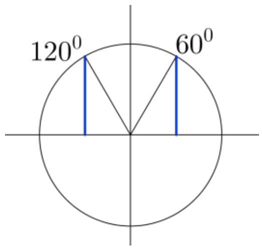


5) $\cos x = -\dfrac{\sqrt{3}}{2} \Rightarrow x = \arccos\left(-\dfrac{\sqrt{3}}{2}\right) \Rightarrow x = \begin{cases} 150^{\circ} + 360^{\circ} k \\ 210^{\circ} + 360^{\circ} k \end{cases} \quad k \in \mathbb{Z}$



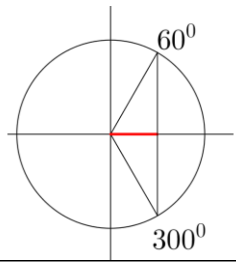
6) $\operatorname{sen} 2x = \dfrac{\sqrt{3}}{2} \Rightarrow 2x = \operatorname{arcsen}\left(\dfrac{\sqrt{3}}{2}\right) \Rightarrow 2x = \begin{cases} 60^{\circ} + 360^{\circ} k \\ 120^{\circ} + 360^{\circ} k \end{cases} \quad k \in \mathbb{Z} \Rightarrow$

$\Rightarrow x = \begin{cases} 30^{\circ} + 180^{\circ} k \\ 60^{\circ} + 180^{\circ} k \end{cases} \quad k \in \mathbb{Z}$



7) $\cos 3x = \dfrac{1}{2} \Rightarrow 3x = \arccos\left(\dfrac{1}{2}\right) \Rightarrow 3x = \begin{cases} 60^{\circ} + 360^{\circ} k \\ 300^{\circ} + 360^{\circ} k \end{cases} \quad k \in \mathbb{Z} \Rightarrow$

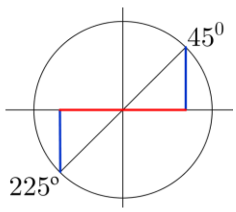
$\Rightarrow x = \begin{cases} 20^{\circ} + 120^{\circ} k \\ 100^{\circ} + 120^{\circ} k \end{cases} \quad k \in \mathbb{Z}$



8) $\operatorname{tg}\left(\dfrac{x}{4}\right) = 1 \Rightarrow \dfrac{x}{4} = \operatorname{arctg}(1) \Rightarrow \dfrac{x}{4} = 45^{\circ} + 180^{\circ} k \quad k \in \mathbb{Z} \Rightarrow$

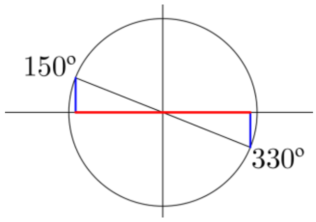
$\Rightarrow x = 180^{\circ} + 720^{\circ} k \quad k \in \mathbb{Z}$ $\Rightarrow$ (expresamos el resultado en radianes)

$\Rightarrow x = \pi + 4\pi \cdot k \quad k \in \mathbb{Z} \Rightarrow x = (4k+1)\pi \quad k \in \mathbb{Z}$



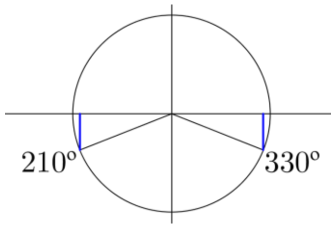
9) $\operatorname{tg} 2x = -\frac{\sqrt{3}}{3} \Rightarrow 2x = \operatorname{arctg}\left(-\frac{\sqrt{3}}{3}\right) \Rightarrow 2x = 150^{o} + 180^{o}k \quad k \in Z \Rightarrow$

$\Rightarrow$ $\boxed{x = 75^{o} + 90^{o}k \quad k \in Z}$



10) $\operatorname{sen} 3x = -\frac{1}{2} \Rightarrow 3x = \operatorname{arcsen}\left(-\frac{1}{2}\right) \Rightarrow 3x = \begin{cases} 210^{o} + 360^{o}k \\ 330^{o} + 360^{o}k \end{cases} \quad k \in Z \Rightarrow$

$\Rightarrow$ $\boxed{x = \begin{cases} 70^{o} + 120^{o}k \\ 110^{o} + 120^{o}k \end{cases} \quad k \in Z}$

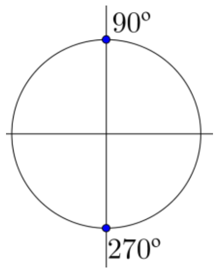


11) $2 \cdot \operatorname{sen} x \cdot \cos x = \cos x$

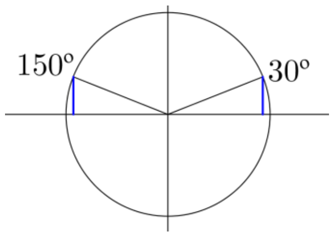
$2 \cdot \operatorname{sen} x \cdot \cos x - \cos x = 0$ (extraemos factor común)

$\cos x \cdot (2\operatorname{sen} x - 1) = 0 \Rightarrow \begin{cases} \underline{\cos x = 0} \\ 2\operatorname{sen} x - 1 = 0 \Rightarrow 2\operatorname{sen} x = 1 \Rightarrow \underline{\operatorname{sen} x = \frac{1}{2}} \end{cases}$

I) $\cos x = 0 \Rightarrow x = \arccos(0) \Rightarrow$ $\boxed{x = 90^{o} + 180^{o}k \quad k \in Z}$



II) $\operatorname{sen} x = \frac{1}{2} \Rightarrow x = \operatorname{arcsen}\left(\frac{1}{2}\right) \Rightarrow$ $\boxed{x = \begin{cases} 30^{o} + 360^{o}k \\ 150^{o} + 360^{o}k \end{cases} \quad k \in Z}$



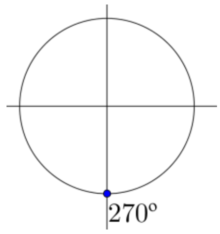
12) $\underbrace{\cos^2 x} - \text{sen}^2 x = \text{sen } x$

$(1-\text{sen}^2 x) - \text{sen}^2 x = \text{sen } x$ (Recuerda: $\text{sen}^2 x + \cos^2 x = 1 \;\Rightarrow\; \cos^2 x = 1 - \text{sen}^2 x$)

$$\boxed{-2\text{sen}^2 x - \text{sen} x + 1 = 0}$$

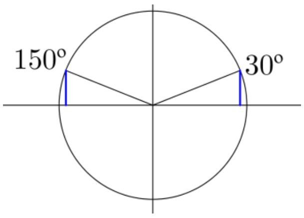
- Hacemos el cambio de variable $\underline{\text{sen } x = t}$ $\Rightarrow$ $-2t^2 - t + 1 = 0$
- Resolvemos la ecuación de 2º grado: $t = \dfrac{1 \pm \sqrt{1+8}}{-4} = \dfrac{1 \pm 3}{-4} = \begin{cases} t = -1 \\ t = \dfrac{1}{2} \end{cases}$
- Deshacemos el cambio de variable:

I) $t = -1 \;\Rightarrow\; \text{sen } x = -1 \;\Rightarrow\; x = \text{arcsen}(-1) \;\Rightarrow\; \boxed{x = 270^\circ + 360^\circ k \quad k \in \mathbb{Z}}$



II) $t = \dfrac{1}{2} \;\Rightarrow\; \text{sen } x = \dfrac{1}{2} \;\Rightarrow\; x = \text{arcsen}\left(\dfrac{1}{2}\right) \;\Rightarrow$

$$\Rightarrow \boxed{x = \begin{cases} 30^\circ + 360^\circ k \\ 150^\circ + 360^\circ k \end{cases} \quad k \in \mathbb{Z}}$$



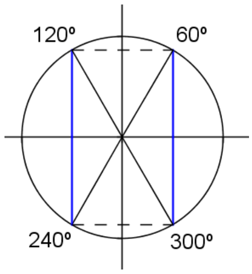
13) $\text{sen}^2 x - \underbrace{\cos^2 x} = \dfrac{1}{2}$

$$\text{sen}^2 x - (1 - \text{sen}^2 x) = \frac{1}{2}$$

$$\text{sen}^2 x - 1 + \text{sen}^2 x = \frac{1}{2} \;\Rightarrow\; 2\text{sen}^2 x = \frac{3}{2} \;\Rightarrow\; \text{sen}^2 x = \frac{3}{4} \;\Rightarrow\; \begin{cases} \text{sen } x = \dfrac{\sqrt{3}}{2} \\ \text{sen } x = -\dfrac{\sqrt{3}}{2} \end{cases}$$

I) $\text{sen } x = \dfrac{\sqrt{3}}{2} \;\Rightarrow\; x = \text{arcsen}\left(\dfrac{\sqrt{3}}{2}\right) \;\Rightarrow\; \boxed{x = \begin{cases} 60^\circ + 360^\circ k \\ 120^\circ + 360^\circ k \end{cases} \quad k \in \mathbb{Z}}$

II) $\text{sen } x = -\dfrac{\sqrt{3}}{2} \;\Rightarrow\; x = \text{arcsen}\left(-\dfrac{\sqrt{3}}{2}\right) \;\Rightarrow\; \boxed{x = \begin{cases} 240^\circ + 360^\circ k \\ 300^\circ + 360^\circ k \end{cases} \quad k \in \mathbb{Z}}$



Podemos expresar las soluciones como, $\boxed{x = \begin{cases} 60^\circ + 180^\circ k \\ 120^\circ + 180^\circ k \end{cases} \quad k \in \mathbb{Z}}$

14) $\text{sen}\, x + \underbrace{\cos^2 x} = \dfrac{5}{4}$

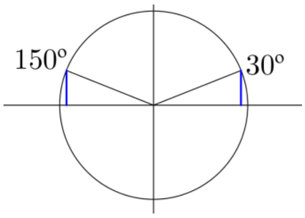
$$\text{sen}\, x + (1 - \text{sen}^2 x) = \frac{5}{4}$$

$$\text{sen}\, x + 1 - \text{sen}^2 x = \frac{5}{4} \quad \Rightarrow \quad -\text{sen}^2 x + \text{sen}\, x + 1 = \frac{5}{4} \quad \underset{(\cdot 4)}{\Rightarrow} \quad -4\text{sen}^2 x + 4\text{sen}\, x + 4 = 5 \quad \Rightarrow$$

$$\Rightarrow \boxed{4\text{sen}^2 x - 4\text{sen}\, x + 1 = 0}$$

- Hacemos el cambio de variable $\text{sen}\, x = t \quad \Rightarrow \quad 4t^2 - 4t + 1 = 0$
- Resolvemos la ecuación de 2º grado: $t = \dfrac{4 \pm \sqrt{16-16}}{8} = \dfrac{4 \pm 0}{8} = \begin{cases} t = \dfrac{1}{2} \\ t = \dfrac{1}{2} \end{cases}$
- Deshacemos el cambio de variable:

$$t = \frac{1}{2} \quad \Rightarrow \quad \text{sen}\, x = \frac{1}{2} \quad \Rightarrow \quad x = \text{arcsen}\left(\frac{1}{2}\right) \quad \Rightarrow$$



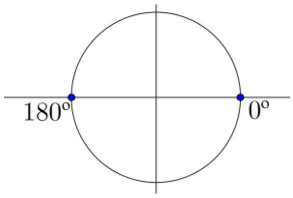
$$\Rightarrow \boxed{x = \begin{cases} 30^\circ + 360^\circ k \\ 150^\circ + 360^\circ k \end{cases} \quad k \in Z}$$

15) $\text{tg}\, x = 2\text{sen}\, x$

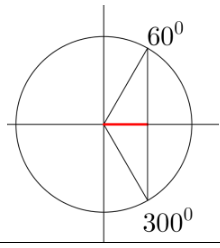
$$\frac{\text{sen}\, x}{\cos x} = 2 \cdot \text{sen}\, x \quad \Rightarrow \quad \text{sen}\, x = 2 \cdot \text{sen}\, x \cdot \cos x \quad \Rightarrow \quad 2 \cdot \text{sen}\, x \cdot \cos x - \text{sen}\, x = 0 \quad \Rightarrow$$

$$\Rightarrow \quad \text{sen}\, x \cdot (2\cos x - 1) = 0 \quad \Rightarrow \quad \begin{cases} \underline{\text{sen}\, x = 0} \\ 2\cos x - 1 = 0 \quad \Rightarrow \quad 2\cos x = 1 \quad \Rightarrow \quad \underline{\cos x = \dfrac{1}{2}} \end{cases}$$

I) $\text{sen}\, x = 0 \quad \Rightarrow \quad x = \text{arcsen}(0) \quad \Rightarrow \quad x = 0^\circ + 180^\circ k \quad k \in Z \quad \Rightarrow$



$$\Rightarrow \boxed{x = 180^\circ k \quad k \in Z}$$

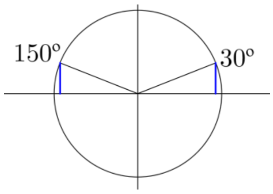


II) $\cos x = \dfrac{1}{2} \quad \Rightarrow \quad x = \arccos\left(\dfrac{1}{2}\right) \quad \Rightarrow \quad \boxed{x = \begin{cases} 60^\circ + 360^\circ k \\ 300^\circ + 360^\circ k \end{cases} \quad k \in Z}$

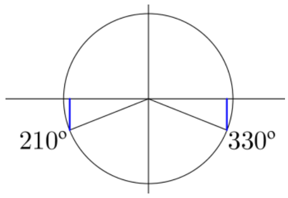
16) $\underbrace{\cos^2 x} - 3\,\text{sen}^2 x = 0$

$$1-\text{sen}^2 x - 3\,\text{sen}^2 x = 0 \;\Rightarrow\; 1-4\,\text{sen}^2 x = 0 \;\Rightarrow\; 1 = 4\,\text{sen}^2 x \;\Rightarrow\; \text{sen}^2 x = \frac{1}{4} \;\Rightarrow\; \begin{cases} \underline{\text{sen}\, x = 1/2} \\ \underline{\text{sen}\, x = -1/2} \end{cases}$$

I) $\text{sen}\, x = \dfrac{1}{2} \;\Rightarrow\; x = \text{arcsen}\left(\dfrac{1}{2}\right) \;\Rightarrow\; \boxed{x = \begin{cases} 30^{\circ} + 360^{\circ} k \\ 150^{\circ} + 360^{\circ} k \end{cases} \quad k \in Z}$



II) $\text{sen}\, x = -\dfrac{1}{2} \;\Rightarrow\; x = \text{arcsen}\left(-\dfrac{1}{2}\right) \;\Rightarrow\; \boxed{x = \begin{cases} 210^{\circ} + 360^{\circ} k \\ 330^{\circ} + 360^{\circ} k \end{cases} \quad k \in Z}$

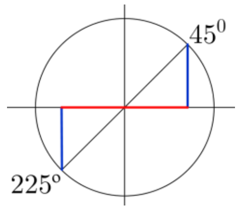


Podemos escribimos las soluciones como , $\boxed{x = \begin{cases} 30^{\circ} + 180^{\circ} k \\ 150^{\circ} + 180^{\circ} k \end{cases} \quad k \in Z}$

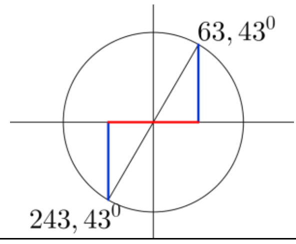
17) $\text{tg}^2 x - 3\,\text{tg}\, x + 2 = 0$

- Hacemos el cambio de variable $\underline{\text{tg}\, x = t} \;\Rightarrow\; t^2 - 3t + 2 = 0$
- Resolvemos la ecuación de 2º grado: $t = \dfrac{3 \pm \sqrt{9-8}}{2} = \dfrac{3 \pm 1}{2} = \begin{cases} t = 2 \\ t = 1 \end{cases}$
- Deshacemos el cambio de variable:

I) $\text{tg}\, x = 1 \;\Rightarrow\; x = \text{arctg}(1) \;\Rightarrow\; \boxed{x = 45^{\circ} + 180^{\circ} k \quad k \in Z}$



II) $\text{tg}\, x = 2 \;\Rightarrow\; x = \text{arctg}(2) \;\Rightarrow\; \boxed{x = 63{,}43^{\circ} + 180^{\circ} k \quad k \in Z}$



18) $\text{tg}\, x \cdot \sec x = \sqrt{2}$

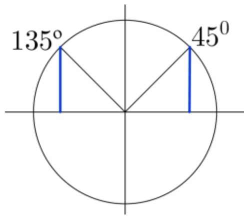
$$\frac{\text{sen}\, x}{\cos x} \cdot \frac{1}{\cos x} = \sqrt{2} \;\Rightarrow\; \frac{\text{sen}\, x}{\cos^2 x} = \sqrt{2} \;\Rightarrow\; \text{sen}\, x = \sqrt{2} \cdot \cos^2 x \;\Rightarrow\; \sqrt{2} \cdot \underbrace{\cos^2 x}_{1-\text{sen}^2 x} - \text{sen}\, x = 0 \;\Rightarrow$$

$$\Rightarrow\; \sqrt{2} \cdot (1 - \text{sen}^2 x) - \text{sen}\, x = 0 \;\Rightarrow\; \sqrt{2} - \sqrt{2}\,\text{sen}^2 x - \text{sen}\, x = 0 \;\Rightarrow\; \boxed{\sqrt{2}\,\text{sen}^2 x + \text{sen}\, x - \sqrt{2} = 0}$$

- Hacemos el cambio de variable $\underline{\text{sen}\, x = t} \;\Rightarrow\; \sqrt{2}\, t^2 + t - \sqrt{2} = 0$
- Resolvemos la ecuación de 2º grado:

$$t=\frac{-1\pm\sqrt{1+8}}{2\sqrt{2}}=\frac{-1\pm 3}{2\sqrt{2}}=\begin{cases} t=\dfrac{2}{2\sqrt{2}}=\dfrac{1}{\sqrt{2}}=\dfrac{\sqrt{2}}{2} \quad \Rightarrow \quad t=\dfrac{\sqrt{2}}{2} \\ t=-\dfrac{4}{2\sqrt{2}}=-\dfrac{2}{\sqrt{2}}=-\dfrac{2\sqrt{2}}{2}=-\sqrt{2} \quad \Rightarrow \quad t=-\sqrt{2} \end{cases}$$

- Deshacemos el cambio de variable:



I) $\operatorname{sen} x=\dfrac{\sqrt{2}}{2} \quad \Rightarrow \quad x=\operatorname{arcsen}\left(\dfrac{\sqrt{2}}{2}\right) \quad \Rightarrow \quad x=\begin{cases} 45^\circ+360^\circ k \\ 135^\circ+360^\circ k \end{cases} \quad k\in Z$

II) $\operatorname{sen} x=-\sqrt{2} \quad \Rightarrow \quad \nexists$ solución ya que $-1\le \operatorname{sen} x\le 1$

19) $\underbrace{\cos^2 x}-\operatorname{sen}^2 x=1+4\operatorname{sen} x$

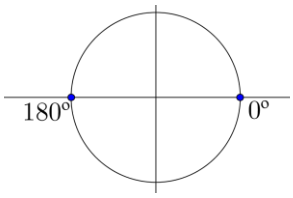
$1-\operatorname{sen}^2 x-\operatorname{sen}^2 x=1+4\operatorname{sen} x$

$1+4\operatorname{sen} x-1+\operatorname{sen}^2 x+\operatorname{sen}^2 x=0$

$2\operatorname{sen}^2 x+4\operatorname{sen} x=0$ (simplificamos :2)

$\operatorname{sen}^2 x+2\operatorname{sen} x=0$ (extraemos factor común)

$\operatorname{sen} x\cdot(\operatorname{sen} x+2)=0 \Rightarrow \begin{cases} \operatorname{sen} x=0 \\ \operatorname{sen} x+2=0 \end{cases}$



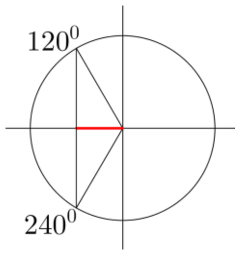
I) $\operatorname{sen} x=0 \quad \Rightarrow \quad x=\operatorname{arcsen}(0) \quad \Rightarrow \quad x=0^\circ+180^\circ k \quad k\in Z \quad \Rightarrow$

$\Rightarrow \quad x=180^\circ k \quad k\in Z$

II) $\operatorname{sen} x+2=0 \quad \Rightarrow \quad \operatorname{sen} x=-2 \quad \Rightarrow \quad \nexists$ solución ya que $-1\le \operatorname{sen} x\le 1$

20) $\cos(4x-\pi)=-\dfrac{1}{2}$

$4x-180^\circ=\arccos\left(-\dfrac{1}{2}\right) \quad \Rightarrow \quad 4x-180^\circ=\begin{cases} 120^\circ+360^\circ k \\ 240^\circ+360^\circ k \end{cases} \quad k\in Z \quad \Rightarrow$



$\Rightarrow \quad 4x=\begin{cases} 300^\circ+360^\circ k \\ 420^\circ+360^\circ k \end{cases} \quad k\in Z \quad \underset{\text{reducimos 420° al primer giro}}{\Rightarrow} \quad 4x=\begin{cases} 300^\circ+360^\circ k \\ 60^\circ+360^\circ k \end{cases} \quad k\in Z \quad \Rightarrow \quad x=\begin{cases} 75^\circ+90^\circ k \\ 15^\circ+90^\circ k \end{cases} \quad k\in Z$

21) $\cos^2 x - \underbrace{\text{sen}^2 x} + 5\cos x + 3 = -1$

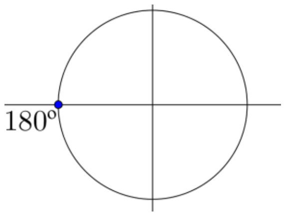
$$\cos^2 x - (1-\cos^2 x) + 5\cos x + 3 + 1 = 0$$

$$\cos^2 x - 1 + \cos^2 x + 5\cos x + 3 + 1 = 0$$

$$\boxed{2\cos^2 x + 5\cos x + 3 = 0}$$

- Hacemos el cambio de variable $\underline{\cos x = t} \quad \Rightarrow \quad 2t^2 + 5t + 3 = 0$
- Resolvemos la ecuación de 2º grado: $t = \dfrac{-5 \pm \sqrt{25-24}}{4} = \dfrac{-5 \pm 1}{4} = \begin{cases} t = -1 \\ t = -\dfrac{3}{2} \end{cases}$
- Deshacemos el cambio de variable:

I) $\cos x = -1 \Rightarrow x = \arccos(-1) \quad \Rightarrow \quad \boxed{x = 180^\circ + 360^\circ k \quad k \in Z}$



II) $\cos x = -\dfrac{3}{2} \Rightarrow \underline{\nexists \text{ solución ya que } -1 \le \cos x \le 1}$

22) $\text{sen } x - \dfrac{1}{\text{sen } x} = -\dfrac{1}{2\sqrt{3}}$

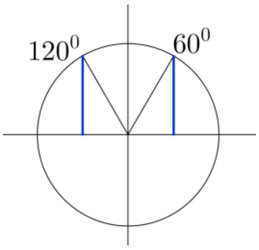
$$\frac{2\sqrt{3}\,\text{sen}^2 x - 2\sqrt{3}}{2\sqrt{3}\,\text{sen } x} = \frac{-\text{sen } x}{2\sqrt{3}\,\text{sen } x} \quad \Rightarrow \quad 2\sqrt{3}\,\text{sen}^2 x - 2\sqrt{3} = -\text{sen } x \quad \Rightarrow \quad \boxed{2\sqrt{3}\,\text{sen}^2 x + \text{sen } x - 2\sqrt{3} = 0}$$

- Hacemos el cambio de variable $\underline{\text{sen } x = t} \quad \Rightarrow \quad 2\sqrt{3}\,t^2 + t - 2\sqrt{3} = 0$
- Resolvemos la ecuación de 2º grado:

$$t = \frac{-1 \pm \sqrt{1+48}}{4\sqrt{3}} = \frac{-1 \pm 7}{4\sqrt{3}} = \begin{cases} t = \dfrac{6}{4\sqrt{3}} = \dfrac{3}{2\sqrt{3}} = \dfrac{3\sqrt{3}}{6} \quad \Rightarrow \quad t = \dfrac{\sqrt{3}}{2} \\ t = -\dfrac{8}{4\sqrt{3}} = -\dfrac{2}{\sqrt{3}} = -\dfrac{2\sqrt{3}}{3} \quad \Rightarrow \quad t = -\dfrac{2\sqrt{3}}{3} \end{cases}$$

- Deshacemos el cambio de variable:

I) $\text{sen } x = \dfrac{\sqrt{3}}{2} \quad \Rightarrow \quad x = \text{arcsen}\left(\dfrac{\sqrt{3}}{2}\right) \quad \Rightarrow \quad \boxed{x = \begin{cases} 60^\circ + 360^\circ k \\ 120^\circ + 360^\circ k \end{cases} \quad k \in Z}$

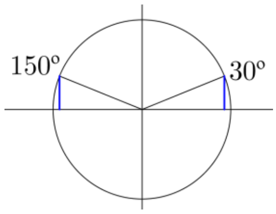


II) $\text{sen } x = -\dfrac{2\sqrt{3}}{3} \quad \Rightarrow \quad \underline{\nexists \text{ solución ya que } -1 \le \text{sen } x \le 1}$

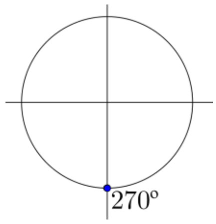
23) $\operatorname{sen} x + 2 = 3\cos^2 x + \operatorname{sen}^2 x$

$$\operatorname{sen} x + 2 = 3(1-\operatorname{sen}^2 x) + \operatorname{sen}^2 x \quad \Rightarrow \quad \operatorname{sen} x + 2 = 3 - 3\operatorname{sen}^2 x + \operatorname{sen}^2 x \quad \Rightarrow \quad \boxed{2\operatorname{sen}^2 x + \operatorname{sen} x - 1 = 0}$$

- Hacemos el cambio de variable $\underline{\operatorname{sen} x = t} \quad \Rightarrow \quad 2t^2 + t - 1 = 0$
- Resolvemos la ecuación de 2º grado: $t = \dfrac{-1 \pm \sqrt{1+8}}{4} = \dfrac{-1 \pm 3}{4} = \begin{cases} t = 1/2 \\ t = -1 \end{cases}$
- Deshacemos el cambio de variable:



I) $\operatorname{sen} x = \dfrac{1}{2} \quad \Rightarrow \quad x = \operatorname{arcsen}\left(\dfrac{1}{2}\right) \quad \Rightarrow \quad \boxed{x = \begin{cases} 30^\circ + 360^\circ k \\ 150^\circ + 360^\circ k \end{cases} \quad k \in Z}$

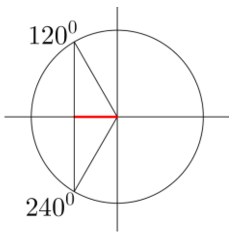


II) $\operatorname{sen} x = -1 \quad \Rightarrow \quad x = \operatorname{arcsen}(-1) \quad \Rightarrow \quad \boxed{x = 270^\circ + 360^\circ k \quad k \in Z}$

24) $\cos^2 x - \operatorname{sen}^2 x + 5\cos x + 3 = 0$

$$\cos^2 x - (1 - \cos^2 x) + 5\cos x + 3 = 0 \quad \Rightarrow \quad \cos^2 x - 1 + \cos^2 x + 5\cos x + 3 = 0 \quad \Rightarrow \quad \boxed{2\cos^2 x + 5\cos x + 2 = 0}$$

- Hacemos el cambio de variable $\underline{\cos x = t} \quad \Rightarrow \quad 2t^2 + 5t + 2 = 0$
- Resolvemos la ecuación de 2º grado: $t = \dfrac{-5 \pm \sqrt{25-16}}{4} = \dfrac{-5 \pm 3}{4} = \begin{cases} t = -\dfrac{1}{2} \\ t = -2 \end{cases}$
- Deshacemos el cambio de variable:



I) $\cos x = -\dfrac{1}{2} \quad \Rightarrow \quad x = \arccos\left(-\dfrac{1}{2}\right) \quad \Rightarrow \quad \boxed{x = \begin{cases} 120^\circ + 360^\circ k \\ 240^\circ + 360^\circ k \end{cases} \quad k \in Z}$

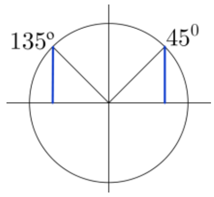
II) $\cos x = -2 \quad \Rightarrow \quad \underline{\nexists \text{ solución ya que } -1 \le \cos x \le 1}$

25) $2\,\text{sen}^4 x - 7\cos^2 x + 3 = 0$

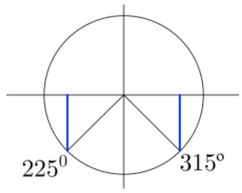
$$2\,\text{sen}^4 x - 7(1-\text{sen}^2 x) + 3 = 0 \;\Rightarrow\; 2\,\text{sen}^4 x - 7 + 7\text{sen}^2 x + 3 = 0 \;\Rightarrow\; \boxed{2\,\text{sen}^4 x + 7\text{sen}^2 x - 4 = 0}$$

- Hacemos el cambio de variable $\underline{\text{sen}\,x = t}$ $\Rightarrow$ $2t^4 + 7t^2 - 4 = 0$ **Ecuación bicuadrada**
- Resolvemos la ecuación bicuadrada: $t^2 = \dfrac{-7 \pm \sqrt{49+32}}{4} = \dfrac{-7 \pm 9}{4} = \begin{cases} t^2 = \dfrac{1}{2} & \Rightarrow\; t = \pm\dfrac{\sqrt{2}}{2} \\ t = -4 & \Rightarrow\; \nexists \text{ solución} \end{cases}$
- Deshacemos el cambio de variable:

I) $\text{sen}\,x = \dfrac{\sqrt{2}}{2} \;\Rightarrow\; x = \text{arcsen}\left(\dfrac{\sqrt{2}}{2}\right) \;\Rightarrow\; \boxed{x = \begin{cases} 45^\circ + 360^\circ k \\ 135^\circ + 360^\circ k \end{cases} \; k \in Z}$



II) $\text{sen}\,x = -\dfrac{\sqrt{2}}{2} \;\Rightarrow\; x = \text{arcsen}\left(-\dfrac{\sqrt{2}}{2}\right) \;\Rightarrow\; \boxed{x = \begin{cases} 225^\circ + 360^\circ k \\ 315^\circ + 360^\circ k \end{cases} \; k \in Z}$

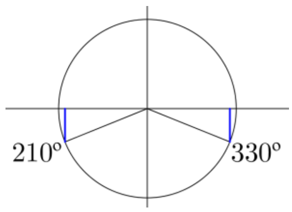


Por tanto, $\boxed{x = \begin{cases} 45^\circ + 180^\circ k \\ 135^\circ + 180^\circ k \end{cases} \; k \in Z}$

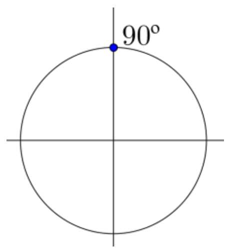
26) $\cos^2 x - \text{sen}^2 x + \text{sen}\,x = 0$

$$1 - \text{sen}^2 x - \text{sen}^2 x + \text{sen}\,x = 0 \;\Rightarrow\; \boxed{2\,\text{sen}^2 x - \text{sen}\,x - 1 = 0}$$

- Hacemos el cambio de variable $\underline{\text{sen}\,x = t}$ $\Rightarrow$ $2t^2 - t - 1 = 0$
- Resolvemos la ecuación de 2º grado: $t = \dfrac{1 \pm \sqrt{1+8}}{4} = \dfrac{1 \pm 3}{4} = \begin{cases} t = 1 \\ t = -1/2 \end{cases}$
- Deshacemos el cambio de variable:



I) $\text{sen}\,x = -\dfrac{1}{2} \;\Rightarrow\; x = \text{arcsen}\left(-\dfrac{1}{2}\right) \;\Rightarrow\; \boxed{x = \begin{cases} 210^\circ + 360^\circ k \\ 330^\circ + 360^\circ k \end{cases} \; k \in Z}$



II) $\text{sen}\,x = 1 \;\Rightarrow\; x = \text{arcsen}(1) \;\Rightarrow\; \boxed{x = 90^\circ + 360^\circ k \quad k \in Z}$

27) $2\cdot \text{sen}\, x \cdot \cos x = \text{tg}\, x$

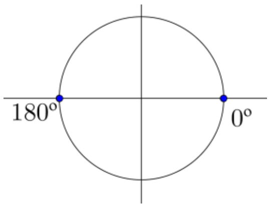
$$2\cdot \text{sen}\, x\cdot\cos x = \frac{\text{sen}\, x}{\cos x} \quad\Rightarrow\quad 2\cdot\text{sen}\, x\cdot\cos^2 x = \text{sen}\, x \quad\Rightarrow\quad 2\cdot\text{sen}\, x\cdot\cos^2 x - \text{sen}\, x = 0 \quad\Rightarrow$$

$$\Rightarrow\quad \text{sen}\, x\cdot(2\cos^2 x - 1) = 0 \quad\Rightarrow\quad \text{sen}\, x\cdot(2\cos^2 x - 1) = 0 \quad\Rightarrow$$

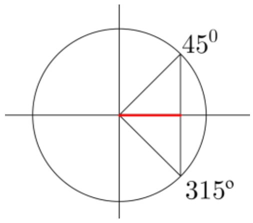
$$\Rightarrow \begin{cases} \underline{\text{sen}\, x = 0} \\ 2\cos^2 x - 1 = 0 \quad\Rightarrow\quad \cos^2 x = \dfrac{1}{2} \quad\Rightarrow\quad \cos x = \pm\dfrac{1}{\sqrt{2}} \quad\Rightarrow\quad \underline{\cos x = \pm\dfrac{\sqrt{2}}{2}} \end{cases}$$

I) $\text{sen}\, x = 0 \quad\Rightarrow\quad x = \text{arcsen}(0) \quad\Rightarrow\quad x = 0^\circ + 180^\circ k \quad k\in Z \quad\Rightarrow$

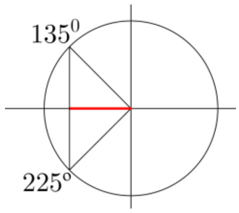
$$\Rightarrow\quad \boxed{x = 180^\circ k \quad k\in Z}$$



II) $\cos x = \dfrac{\sqrt{2}}{2} \quad\Rightarrow\quad x = \arccos\left(\dfrac{\sqrt{2}}{2}\right) \quad\Rightarrow\quad \boxed{x = \begin{cases} 45^\circ + 360^\circ k \\ 315^\circ + 360^\circ k \end{cases} \quad k\in Z}$



III) $\cos x = -\dfrac{\sqrt{2}}{2} \quad\Rightarrow\quad x = \arccos\left(-\dfrac{\sqrt{2}}{2}\right) \quad\Rightarrow\quad \boxed{x = \begin{cases} 135^\circ + 360^\circ k \\ 225^\circ + 360^\circ k \end{cases} \quad k\in Z}$



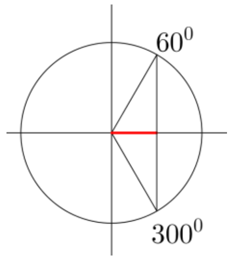
Las soluciones de II y III se pueden expresar como , $\boxed{x = \begin{cases} 45^\circ + 180^\circ k \\ 135^\circ + 180^\circ k \end{cases} \quad k\in Z}$

28) $\cos^2 x - \text{sen}^2 x + \cos x = 0$

$$\cos^2 x - (1-\cos^2 x) + \cos x = 0 \quad\Rightarrow\quad \cos^2 x - 1 + \cos^2 x + \cos x = 0 \quad\Rightarrow\quad \boxed{2\cos^2 x + \cos x - 1 = 0}$$

- Hacemos el cambio de variable $\underline{\cos x = t}$ $\quad\Rightarrow\quad 2t^2 + t - 1 = 0$
- Resolvemos la ecuación de 2º grado: $t = \dfrac{-1\pm\sqrt{1+8}}{4} = \dfrac{-1\pm 3}{4} = \begin{cases} t = \dfrac{1}{2} \\ t = -1 \end{cases}$
- Deshacemos el cambio de variable:

I) $\cos x = \dfrac{1}{2} \quad\Rightarrow\quad x = \arccos\left(\dfrac{1}{2}\right) \quad\Rightarrow\quad \boxed{x = \begin{cases} 60^\circ + 360^\circ k \\ 300^\circ + 360^\circ k \end{cases} \quad k\in Z}$



II) $\cos x = -1 \quad\Rightarrow\quad x = \arccos(-1) \quad\Rightarrow\quad \boxed{x = 180^\circ + 360^\circ k \quad k\in Z}$