<u>Ejercicio 4</u>: Calcula los siguientes límites

1) $$\lim_{x\to+\infty}\frac{2x^2-14x+12}{x^2-10x+4}=\frac{+\infty}{+\infty}(\text{I})=\lim_{x\to+\infty}\frac{\frac{2x^2}{x^2}-\frac{14x}{x^2}+\frac{12}{x^2}}{\frac{x^2}{x^2}-\frac{10x}{x^2}+\frac{4}{x^2}}=\lim_{x\to+\infty}\frac{2-\frac{14}{x}+\frac{12}{x^2}}{1-\frac{10}{x}+\frac{4}{x^2}}=\frac{2-0+0}{1-0+0}=\frac{2}{1}=2$$

2) $$\lim_{x\to+\infty}\frac{(2x^2-3)(5x-4)}{2x^3-2x^2+1}=\lim_{x\to+\infty}\frac{10x^3-8x^2-15x+12}{2x^3-2x^2+1}=\frac{+\infty}{+\infty}(\text{I})=$$

$$=\lim_{x\to+\infty}\frac{\frac{10x^3}{x^3}-\frac{8x^2}{x^3}-\frac{15x}{x^3}+\frac{12}{x^3}}{\frac{2x^3}{x^3}-\frac{2x^2}{x^3}+\frac{1}{x^3}}=\lim_{x\to+\infty}\frac{10-\frac{8}{x}-\frac{15}{x^2}+\frac{12}{x^3}}{2-\frac{2}{x}+\frac{1}{x^3}}=\frac{10-0-0+0}{2-0+0}=5$$

3) $$\lim_{x\to-1}\frac{x^3+1}{x^2+2x+1}=\frac{0}{0}(\text{I})=\lim_{x\to-1}\frac{(x+1)(x^2-x+1)}{(x+1)^2}=\lim_{x\to-1}\frac{x^2-x+1}{x+1}=\frac{3}{0}=\begin{cases}\lim_{x\to-1^-}\frac{x^2-x+1}{x+1}=\frac{3}{0^-}=-\infty\\ \lim_{x\to-1^+}\frac{x^2-x+1}{x+1}=\frac{3}{0^+}=+\infty\end{cases}$$

	1	0	0	+1
−1		−1	+1	−1
	1	−1	+1	0

$\Rightarrow\ x^3+1=(x-1)(x^2-x+1)$

$$x^2+2x+1\underset{\substack{\text{identidad}\\ \text{notable}}}{=}(x+1)^2$$

4) $$\lim_{x\to-1}\frac{x^2-1}{x^2+3x+2}=\frac{0}{0}(\text{I})=\lim_{x\to-1}\frac{(x+1)(x-1)}{(x+1)(x+2)}=\lim_{x\to-1}\frac{x-1}{x+2}=\frac{-2}{1}=-2$$

	1	+3	+2
−1		−1	−2
	1	+2	0

$\Rightarrow x^2+3x+2=(x+1)(x+2)$

5) $$\lim_{x\to a}\frac{3x^2-2ax-a^2}{2x^2-3ax+a^2}=\frac{0}{0}(\text{I})=\lim_{x\to a}\frac{(x-a)(3x+a)}{(x-a)(2x-a)}=\lim_{x\to a}\frac{3x+a}{2x-a}=\frac{3a+a}{2a-a}=\frac{4a}{a}=4\qquad(a\neq 0)$$

$$\begin{array}{c|ccc} & 3 & -2a & -a^2 \\ a & & +3a & +a^2 \\ \hline & 3 & +a & 0 \end{array} \Rightarrow 3x^2-2ax-a^2=(x-a)(3x+a)$$

$$\begin{array}{c|ccc} & 2 & -3a & +a^2 \\ a & & +2a & -a^2 \\ \hline & 2 & -a & 0 \end{array} \Rightarrow 2x^2-3ax+a^2=(x-a)(2x-a)$$

6) $$\lim_{x\to 1}\left(\frac{1}{1-x}-\frac{3}{1-x^2}\right)=\infty-\infty(\mathrm{I})=\lim_{x\to 1}\left(\frac{1}{1-x}-\frac{3}{(1-x)(1+x)}\right)=\lim_{x\to 1}\frac{1+x-3}{(1-x)(1+x)}=\lim_{x\to 1}\frac{x-2}{(1-x)(1+x)}=\frac{-1}{0}=$$

$$=\begin{cases}\lim\limits_{x\to 1^-}\dfrac{x-2}{(1-x)(1+x)}=\dfrac{-1}{(0^+)(2)}=\dfrac{-1}{0^+}=-\infty \\ \lim\limits_{x\to 1^+}\dfrac{x-2}{(1-x)(1+x)}=\dfrac{-1}{(0^-)(2)}=\dfrac{-1}{0^-}=+\infty\end{cases}$$

7) $$\lim_{x\to+\infty}\frac{\sqrt{x+4}}{x-2}=\frac{+\infty}{+\infty}(\mathrm{I})\underset{\text{dos formas}}{=}\begin{cases}\lim\limits_{x\to+\infty}\dfrac{\sqrt{\dfrac{x}{x^2}+\dfrac{4}{x^2}}}{\dfrac{x}{x^2}-\dfrac{2}{x^2}}=\lim\limits_{x\to+\infty}\dfrac{\sqrt{\dfrac{1}{x}+\dfrac{4}{x^2}}}{1-\dfrac{2}{x}}=\dfrac{\sqrt{0+0}}{1-0}=\dfrac{0}{1}=0 \\ \lim\limits_{x\to+\infty}\dfrac{\sqrt{x+4}}{x-2}=\lim\limits_{x\to+\infty}\dfrac{\sqrt{x}}{x}=\lim\limits_{x\to+\infty}\dfrac{x^{\frac{1}{2}}}{x}=\lim\limits_{x\to+\infty}x^{-\frac{1}{2}}=\lim\limits_{x\to+\infty}\dfrac{1}{x^{\frac{1}{2}}}=\lim\limits_{x\to+\infty}\dfrac{1}{\sqrt{x}}=\dfrac{1}{+\infty}=0\end{cases}$$

8) $$\lim_{x\to 2}\frac{\sqrt{x+2}-2}{\sqrt{x+7}-3}=\frac{0}{0}(\mathrm{I})=\lim_{x\to 2}\frac{[(\sqrt{x+2}-2)(\sqrt{x+2}+2)](\sqrt{x+7}+3)}{[(\sqrt{x+7}-3)(\sqrt{x+7}+3)](\sqrt{x+2}+2)}=$$

$$=\lim_{x\to 2}\frac{[(\sqrt{x+2})^2-(2)^2](\sqrt{x+7}+3)}{[(\sqrt{x+7})^2-(3)^2](\sqrt{x+2}+2)}=\lim_{x\to 2}\frac{(x+2-4)(\sqrt{x+7}+3)}{(x+7-9)(\sqrt{x+2}+2)}=\lim_{x\to 2}\frac{(x-2)(\sqrt{x+7}+3)}{(x-2)(\sqrt{x+2}+2)}=$$

$$=\lim_{x\to 2}\frac{(\sqrt{x+7}+3)}{(\sqrt{x+2}+2)}=\frac{\sqrt{2+7}+3}{\sqrt{2+2}+2}=\frac{3+3}{2+2}=\frac{6}{4}=\frac{3}{2}$$

9) $$\lim_{x\to 3}\frac{9-x^2}{\sqrt{3x}-3}=\frac{0}{0}(\mathrm{I})=\lim_{x\to 3}\frac{(9-x^2)(\sqrt{3x}+3)}{(\sqrt{3x}-3)(\sqrt{3x}+3)}=\lim_{x\to 3}\frac{(9-x^2)(\sqrt{3x}+3)}{(\sqrt{3x})^2-(3)^2}=\lim_{x\to 3}\frac{(9-x^2)(\sqrt{3x}+3)}{3x-9}\underset{\substack{\text{factorizar}\\\text{polinomios}}}{=}$$

$$=\lim_{x\to 3}\frac{(3-x)(3+x)(\sqrt{3x}+3)}{3(x-3)}=\lim_{x\to 3}\frac{-(x-3)(3+x)(\sqrt{3x}+3)}{3(x-3)}=\lim_{x\to 3}\frac{-(3+x)(\sqrt{3x}+3)}{3}=\frac{-6\cdot 6}{3}=-12$$

10) $\lim_{x\to+\infty}\left(\sqrt{x(x+1)}-x\right)=\lim_{x\to+\infty}\left(\sqrt{x^2+x}-x\right)=\lim_{x\to+\infty}\frac{(\sqrt{x^2+x}-x)(\sqrt{x^2+x}+x)}{\sqrt{x^2+x}+x}=\lim_{x\to+\infty}\frac{(\sqrt{x^2+x})^2-(x)^2}{\sqrt{x^2+x}-x}=$

$=\lim_{x\to+\infty}\frac{x^2+x-x^2}{\sqrt{x^2+x}+x}=\lim_{x\to+\infty}\frac{x}{\sqrt{x^2+x}+x}=\frac{+\infty}{+\infty}(\text{I})=$

$=\lim_{x\to+\infty}\frac{x}{\sqrt{x^2+x}+x}=\lim_{x\to+\infty}\frac{\frac{x}{x}}{\sqrt{\frac{x^2}{x^2}+\frac{x}{x^2}}+\frac{x}{x}}=\lim_{x\to+\infty}\frac{1}{\sqrt{1+\frac{1}{x}}+1}=\frac{1}{\sqrt{1+0}+1}=\frac{1}{2}$

11) $\lim_{x\to4}\left(\frac{x+6}{x^2-16}-\frac{x+1}{x^2-4x}\right)=\infty-\infty(\text{I})=\lim_{x\to4}\left(\frac{x+6}{(x-4)(x+4)}-\frac{x+1}{x(x-4)}\right)=\lim_{x\to4}\left(\frac{x(x+6)-(x+4)(x+1)}{x(x-4)(x+4)}\right)=$

$=\lim_{x\to4}\left(\frac{x^2+6x-x^2-x-4x-4}{x(x-4)(x+4)}\right)=\lim_{x\to4}\frac{x-4}{x(x-4)(x+4)}=\lim_{x\to4}\frac{1}{x(x+4)}=\frac{1}{32}$

12) $\lim_{x\to0}\left(\frac{x^2+3}{x^3}-\frac{1}{x}\right)=\infty-\infty(\text{I})=\lim_{x\to0}\left(\frac{x^2+3-x^2}{x^3}\right)=\lim_{x\to0}\left(\frac{3}{x^3}\right)=\frac{3}{0}=\begin{cases}\lim_{x\to0^-}\frac{3}{x^3}=\frac{3}{0^-}=-\infty\\ \lim_{x\to0^+}\frac{3}{x^3}=\frac{3}{0^+}=+\infty\end{cases}$

13) $\lim_{x\to1}\left(\frac{2}{(x-1)^2}-\frac{1}{x(x-1)}\right)=\infty-\infty(\text{I})=\lim_{x\to1}\left(\frac{2x-(x-1)}{x(x-1)^2}\right)=\lim_{x\to1}\frac{2x-x+1}{x(x-1)^2}=\lim_{x\to1}\frac{x+1}{x(x-1)^2}=\frac{2}{0}=$

$=\begin{cases}\lim_{x\to1^-}\frac{x+1}{x(x-1)^2}=\frac{2}{1\cdot(0^-)^2}=\frac{2}{0^+}=+\infty\\ \lim_{x\to1^+}\frac{x+1}{x(x-1)^2}=\frac{2}{1\cdot(0^+)^2}=\frac{2}{0^+}=+\infty\end{cases}$

14) $\lim_{x\to+\infty}\left(\sqrt{\frac{x}{1+2x}}\right)^{\frac{x}{x+1}}=\lim_{x\to+\infty}\left(\left(\frac{x}{1+2x}\right)^{\frac{1}{2}}\right)^{\frac{x}{x+1}}=\lim_{x\to+\infty}\left(\frac{x}{1+2x}\right)^{\frac{x}{2x+2}}=\left[\lim_{x\to+\infty}\left(\frac{x}{1+2x}\right)\right]^{\lim_{x\to+\infty}\left(\frac{x}{2x+2}\right)}=$

$=\left(\frac{1}{2}\right)^{\frac{1}{2}}=\sqrt{\frac{1}{2}}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$

- $\lim_{x\to+\infty}\left(\frac{x}{1+2x}\right)=\frac{+\infty}{+\infty}=\lim_{x\to+\infty}\frac{\frac{x}{x}}{\frac{1}{x}+\frac{2x}{x}}=\lim_{x\to+\infty}\frac{1}{\frac{1}{x}+2}=\frac{1}{0+2}=\frac{1}{2}$
- $\lim_{x\to+\infty}\left(\frac{x}{2x+2}\right)=\frac{+\infty}{+\infty}=\lim_{x\to+\infty}\frac{\frac{x}{x}}{\frac{2x}{x}+\frac{2}{x}}=\lim_{x\to+\infty}\frac{1}{2+\frac{2}{x}}=\frac{1}{2+0}=\frac{1}{2}$

15) $\lim_{x\to 2}\left(\frac{3}{x+1}\right)^{\frac{x^2+2x+5}{x}} = 1^{\frac{13}{2}} = 1$

16) $\lim_{x\to+\infty}\left(\frac{x+1}{x-1}\right)^{x} = 1^{\infty}\,(\text{I}) = \lim_{x\to+\infty}\left(1+\underbrace{\frac{x+1}{x-1}-1}_{\text{operamos}}\right)^{x} = \lim_{x\to+\infty}\left(1+\frac{2}{x-1}\right)^{x} = \lim_{x\to+\infty}\left(1+\frac{1}{\frac{x-1}{2}}\right)^{x} =$

$= \lim_{x\to+\infty}\left(1+\frac{1}{\frac{x-1}{2}}\right)^{\frac{x-1}{2}\cdot\frac{2}{x-1}\cdot x} = \lim_{x\to+\infty}\left[\left(1+\frac{1}{\frac{x-1}{2}}\right)^{\frac{x-1}{2}}\right]^{\frac{2}{x-1}\cdot x} = \left[\lim_{x\to+\infty}\left(1+\frac{1}{\frac{x-1}{2}}\right)^{\frac{x-1}{2}}\right]^{\lim\limits_{x\to+\infty}\frac{2}{x-1}\cdot x} = e^{\lim\limits_{x\to+\infty}\frac{2x}{x-1}} \underset{(*)}{=} e^2$

(*) $\lim_{x\to+\infty}\frac{2x}{x-1} = \frac{-\infty}{+\infty} = (\text{I}) = \lim_{x\to+\infty}\frac{\frac{2x}{x}}{\frac{x}{x}-\frac{1}{x}} = \lim_{x\to+\infty}\frac{2}{1-\frac{1}{x}} = \frac{2}{1-0} = 2$

17) $\lim_{x\to 1}\left(\frac{x^2+2x+1}{x+3}\right)^{\frac{x^3+2x^2+5}{x^2+4x-5}} = 1^{\infty}\,(\text{I}) = \lim_{x\to 1}\left(1+\underbrace{\frac{x^2+2x+1}{x+3}-1}_{\text{operamos}}\right)^{\frac{x^3+2x^2+5}{x^2+4x-5}} =$

$= \lim_{x\to 1}\left(1+\frac{x^2+x-2}{x+3}\right)^{\frac{x^3+2x^2+5}{x^2+4x-5}} = \lim_{x\to 1}\left(1+\frac{1}{\frac{x+3}{x^2+x-2}}\right)^{\frac{x^3+2x^2+5}{x^2+4x-5}} =$

$= \lim_{x\to 1}\left(1+\frac{1}{\frac{x+3}{x^2+x-2}}\right)^{\frac{x+3}{x^2+x-2}\cdot\frac{x^2+x-2}{x+3}\cdot\frac{x^3+2x^2+5}{x^2+4x-5}} = \lim_{x\to 1}\left[\left(1+\frac{1}{\frac{x+3}{x^2+x-2}}\right)^{\frac{x+3}{x^2+x-2}}\right]^{\frac{x^2+x-2}{x+3}\cdot\frac{x^3+2x^2+5}{x^2+4x-5}} =$

$= \left[\lim_{x\to 1}\left(1+\frac{1}{\frac{x+3}{x^2+x-2}}\right)^{\frac{x+3}{x^2+x-2}}\right]^{\lim\limits_{x\to 1}\frac{x^2+x-2}{x+3}\cdot\frac{x^3+2x^2+5}{x^2+4x-5}} = e^{\lim\limits_{x\to 1}\frac{x^2+x-2}{x+3}\cdot\frac{x^3+2x^2+5}{x^2+4x-5}} \underset{(*)}{=} e^1 = e$

$$\underset{(*)}{=}\lim_{x\to 1}\frac{x^2+x-2}{x+3}\cdot\frac{x^3+2x^2+5}{x^2+4x-5}=0\cdot\infty\ (\mathrm{I})=\lim_{x\to 1}\frac{(x^2+x-2)(x^3+2x^2+5)}{(x+3)(x^2+4x-5)}=\frac{0}{0}\ (\mathrm{I})=$$

$$=\lim_{x\to 1}\frac{(x-1)(x+2)(x^3+2x^2+5)}{(x+3)(x-1)(x+5)}=\frac{0}{0}\ (\mathrm{I})=\lim_{x\to 1}\frac{(x+2)(x^3+2x^2+5)}{(x+3)(x+5)}=\frac{3\cdot 8}{4\cdot 6}=1$$

18) $$\lim_{x\to+\infty}\left(1-\frac{1}{x}\right)^{x^2}=1^{\infty}(\mathrm{I})=\lim_{x\to+\infty}\left(1+\frac{1}{-x}\right)^{x^2}=\lim_{x\to+\infty}\left(1+\frac{1}{-x}\right)^{(-x)\cdot(-x)}=\lim_{x\to+\infty}\left[\left(1+\frac{1}{-x}\right)^{(-x)}\right]^{(-x)}=$$

$$=\left[\lim_{x\to+\infty}\left(1+\frac{1}{-x}\right)^{(-x)}\right]^{\lim_{x\to+\infty}(-x)}=e^{\lim_{x\to+\infty}(-x)}=e^{-\infty}=0$$

19) $$\lim_{x\to+\infty}\left(\sqrt{x^3-x}-\sqrt{x^3-2x^2}\right)=\infty-\infty(\mathrm{I})=\lim_{x\to+\infty}\frac{(\sqrt{x^3-x}-\sqrt{x^3-2x^2})(\sqrt{x^3-x}+\sqrt{x^3-2x^2})}{(\sqrt{x^3-x}+\sqrt{x^3-2x^2})}=$$

$$=\lim_{x\to+\infty}\frac{(\sqrt{x^3-x})^2-(\sqrt{x^3-2x^2})^2}{(\sqrt{x^3-x}+\sqrt{x^3-2x^2})}=\lim_{x\to+\infty}\frac{x^3-x-(x^3-2x^2)}{(\sqrt{x^3-x}+\sqrt{x^3-2x^2})}=\lim_{x\to+\infty}\frac{x^3-x-x^3+2x^2}{(\sqrt{x^3-x}+\sqrt{x^3-2x^2})}=$$

$$=\lim_{x\to+\infty}\frac{2x^2-x}{(\sqrt{x^3-x}+\sqrt{x^3-2x^2})}=\frac{+\infty}{+\infty}(\mathrm{I})=$$

$$\underset{\text{dos formas distintas}}{=}\begin{cases}\lim_{x\to+\infty}\dfrac{\frac{2x^2}{x^2}-\frac{x}{x^2}}{\sqrt{\frac{x^3}{x^4}-\frac{x}{x^4}}+\sqrt{\frac{x^3}{x^4}-\frac{2x^2}{x^4}}}=\lim_{x\to+\infty}\dfrac{2-\frac{1}{x}}{\sqrt{\frac{1}{x}-\frac{1}{x^3}}+\sqrt{\frac{1}{x}-\frac{2}{x^2}}}=\dfrac{2-0}{\sqrt{0-0}+\sqrt{0+0}}=\dfrac{2}{0}=+\infty\\ \lim_{x\to+\infty}\dfrac{2x^2}{\sqrt{x^3}+\sqrt{x^3}}=\lim_{x\to+\infty}\dfrac{2x^2}{2\sqrt{x^3}}=\lim_{x\to+\infty}\dfrac{x^2}{\sqrt{x^3}}=\lim_{x\to+\infty}\dfrac{x^2}{x^{\frac{3}{2}}}=\lim_{x\to+\infty}x^{\frac{1}{2}}=\lim_{x\to+\infty}\sqrt{x}=+\infty\end{cases}$$

20) $$\lim_{x\to+\infty}\frac{3^x}{2^x}=\lim_{x\to+\infty}\left(\frac{3}{2}\right)^x=\left(\frac{3}{2}\right)^{+\infty}=+\infty$$

21) $$\lim_{x\to+\infty}\frac{2^x}{3^x}=\lim_{x\to+\infty}\left(\frac{2}{3}\right)^x=\left(\frac{2}{3}\right)^{+\infty}=0$$

22) $$\lim_{x\to+\infty}\left(\frac{2x+5}{2x}\right)^{\frac{3x^2-1}{x+2}}=1^\infty(\text{I})=\lim_{x\to+\infty}\left(1+\underbrace{\frac{2x+5}{2x}-1}_{\text{operamos}}\right)^{\frac{3x^2-1}{x+2}}=\lim_{x\to+\infty}\left(1+\frac{5}{2x}\right)^{\frac{3x^2-1}{x+2}}=\lim_{x\to+\infty}\left(1+\frac{1}{\frac{2x}{5}}\right)^{\frac{3x^2-1}{x+2}}=$$

$$=\lim_{x\to+\infty}\left(1+\frac{1}{\frac{2x}{5}}\right)^{\frac{2x}{5}\cdot\frac{5}{2x}\cdot\frac{3x^2-1}{x+2}}=\lim_{x\to+\infty}\left[\left(1+\frac{1}{\frac{2x}{5}}\right)^{\frac{2x}{5}}\right]^{\frac{5}{2x}\cdot\frac{3x^2-1}{x+2}}=\left[\lim_{x\to+\infty}\left(1+\frac{1}{\frac{2x}{5}}\right)^{\frac{2x}{5}}\right]^{\lim_{x\to+\infty}\frac{5}{2x}\cdot\frac{3x^2-1}{x+2}}=$$

$$e^{\lim_{x\to+\infty}\frac{5}{2x}\cdot\frac{3x^2-1}{x+2}}\underset{(*)}{=}e^{\frac{15}{2}}=\sqrt{e^{15}}=e^7\sqrt{e}$$

$$(*)=\lim_{x\to+\infty}\frac{5}{2x}\cdot\frac{3x^2-1}{x+2}=\lim_{x\to+\infty}\frac{15x^2-5}{2x^2+4x}=\frac{+\infty}{+\infty}=\lim_{x\to+\infty}\frac{\frac{15x^2}{x^2}-\frac{5}{x^2}}{\frac{2x^2}{x^2}+\frac{4x}{x^2}}=\lim_{x\to+\infty}\frac{15-\frac{5}{x^2}}{2+\frac{4}{x}}=\frac{15-0}{2+0}=\frac{15}{2}$$

23) $$\lim_{x\to+\infty}\frac{\sqrt{x+a}+\sqrt{x+b}}{\sqrt{x}}=\frac{+\infty}{+\infty}(\text{I})=\begin{cases}\circ\ \lim_{x\to+\infty}\frac{\sqrt{x+a}+\sqrt{x+b}}{\sqrt{x}}=\lim_{x\to+\infty}\frac{\sqrt{\frac{x}{x}+\frac{a}{x}}+\sqrt{\frac{x}{x}+\frac{b}{x}}}{\sqrt{\frac{x}{x}}}==\lim_{x\to+\infty}\frac{\sqrt{1+\frac{a}{x}}+\sqrt{1+\frac{b}{x}}}{\sqrt{1}}=\\=\frac{\sqrt{1+0}+\sqrt{1+0}}{\sqrt{1}}=\frac{1+1}{1}=2\\\circ\ \lim_{x\to+\infty}\frac{\sqrt{x}+\sqrt{x}}{\sqrt{x}}=\lim_{x\to+\infty}\frac{2\sqrt{x}}{\sqrt{x}}=\lim_{x\to+\infty}2=2\end{cases}$$

24) $$\lim_{x\to+\infty}\left(\sqrt[x-3]{\frac{3x+1}{3x+5}}\right)^{x^2-2x}=\lim_{x\to+\infty}\left(\left(\frac{3x+1}{3x+5}\right)^{\frac{1}{x-3}}\right)^{x^2-2x}=\lim_{x\to+\infty}\left(\frac{3x+1}{3x+5}\right)^{\frac{x^2-2x}{x-3}}=1^\infty(\text{I})=$$

$$=\lim_{x\to+\infty}\left(\frac{3x+1}{3x+5}\right)^{\frac{x^2-2x}{x-3}}=\lim_{x\to+\infty}\left(1+\underbrace{\frac{3x+1}{3x+5}-1}_{\text{operamos}}\right)^{\frac{x^2-2x}{x-3}}=\lim_{x\to+\infty}\left(1+\frac{-4}{3x+5}\right)^{\frac{x^2-2x}{x-3}}=\lim_{x\to+\infty}\left(1+\frac{1}{\frac{3x+5}{-4}}\right)^{\frac{x^2-2x}{x-3}}=$$

$$=\lim_{x\to+\infty}\left(1+\frac{1}{\frac{3x+5}{-4}}\right)^{\frac{x^2-2x}{x-3}}=\lim_{x\to+\infty}\left(1+\frac{1}{\frac{3x+5}{-4}}\right)^{\frac{3x+5}{-4}\cdot\frac{-4}{3x+5}\cdot\frac{x^2-2x}{x-3}}=\lim_{x\to+\infty}\left[\left(1+\frac{1}{\frac{3x+5}{-4}}\right)^{\frac{3x+5}{-4}}\right]^{\frac{-4}{3x+5}\cdot\frac{x^2-2x}{x-3}}=$$

$$=\left[\lim_{x\to+\infty}\left(1+\frac{1}{\frac{3x+5}{-4}}\right)^{\frac{3x+5}{-4}}\right]^{\lim\limits_{x\to+\infty}\frac{-4}{3x+5}\cdot\frac{x^2-2x}{x-3}} = e^{\lim\limits_{x\to+\infty}\frac{-4x^2+8x}{3x^2-4x-15}} \underset{(*)}{=} e^{-\frac{4}{3}}$$

$$(*)\lim_{x\to+\infty}\left(\frac{-4x^2+8x}{3x^2-4x-15}\right)=\frac{-\infty}{+\infty}=\lim_{x\to+\infty}\frac{\frac{-4x^2}{x^2}+\frac{8x}{x^2}}{\frac{3x^2}{x^2}-\frac{4x}{x^2}-\frac{15}{x^2}}=\lim_{x\to+\infty}\frac{-4+\frac{8}{x}}{3-\frac{4}{x}-\frac{15}{x^2}}=\frac{-4+0}{3-0-0}=-\frac{4}{3}$$

25) $$\lim_{x\to+\infty}\frac{2x^2-3x-4}{\sqrt{x^4+1}}=\frac{+\infty}{+\infty}=\begin{cases}\lim\limits_{x\to+\infty}\frac{2x^2-3x-4}{\sqrt{x^4+1}}=\lim\limits_{x\to+\infty}\frac{\frac{2x^2}{x^2}-\frac{3x}{x^2}-\frac{4}{x^2}}{\sqrt{\frac{x^4}{x^4}+\frac{1}{x^4}}}=\lim\limits_{x\to+\infty}\frac{2-\frac{3}{x}-\frac{4}{x^2}}{\sqrt{1+\frac{1}{x^4}}}=\frac{2-0-0}{\sqrt{1+0}}=\frac{2}{1}=2\\ \lim\limits_{x\to+\infty}\frac{2x^2}{\sqrt{x^4}}=\lim\limits_{x\to+\infty}\frac{2x^2}{x^2}=\lim\limits_{x\to+\infty}2=2\end{cases}$$

26) $$\lim_{x\to+\infty}\frac{x^2}{10+x\sqrt{x}}=\lim_{x\to+\infty}\frac{x^2}{10+\sqrt{x^3}}=\frac{+\infty}{+\infty}(\text{I})=\begin{cases}\lim\limits_{x\to+\infty}\frac{x^2}{10+\sqrt{x^3}}=\lim\limits_{x\to+\infty}\frac{\frac{x^2}{x^2}}{\frac{10}{x^2}+\sqrt{\frac{x^3}{x^4}}}=\lim\limits_{x\to+\infty}\frac{1}{\frac{10}{x^2}+\sqrt{\frac{1}{x}}}=\frac{1}{0}=+\infty\\ \lim\limits_{x\to+\infty}\frac{x^2}{\sqrt{x^3}}=\lim\limits_{x\to+\infty}\frac{x^2}{x^{\frac{3}{2}}}=\lim\limits_{x\to+\infty}x^{\frac{1}{2}}=\lim\limits_{x\to+\infty}\sqrt{x}=+\infty\end{cases}$$

27) $$\lim_{x\to+\infty}\frac{\sqrt[3]{x^2+1}}{x+1}=\frac{+\infty}{+\infty}(\text{I})=\begin{cases}\lim\limits_{x\to+\infty}\frac{\sqrt[3]{x^2+1}}{x+1}=\lim\limits_{x\to+\infty}\frac{\sqrt[3]{\frac{x^2}{x^3}+\frac{1}{x^3}}}{\frac{x}{x}+\frac{1}{x}}=\lim\limits_{x\to+\infty}\frac{\sqrt[3]{\frac{1}{x}+\frac{1}{x^3}}}{1+\frac{1}{x}}=\frac{\sqrt[3]{0+0}}{1+0}=\frac{0}{1}=0\\ \lim\limits_{x\to+\infty}\frac{\sqrt[3]{x^2}}{x}=\lim\limits_{x\to+\infty}\frac{x^{\frac{2}{3}}}{x}=\lim\limits_{x\to+\infty}x^{-\frac{1}{3}}=\lim\limits_{x\to+\infty}\frac{1}{x^{\frac{1}{3}}}=\lim\limits_{x\to+\infty}\frac{1}{\sqrt[3]{x}}=\frac{1}{+\infty}=0\end{cases}$$

28) $$\lim_{x\to a}\frac{x^2-(a+1)x+a}{x^3-a^3}=\frac{0}{0}(\text{I})=\lim_{x\to a}\frac{(x-a)(x-1)}{(x-a)(x^2+ax+a^2)}=\lim_{x\to a}\frac{(x-1)}{(x^2+ax+a^2)}=\frac{a-1}{a^2+a^2+a^2}=\frac{a-1}{3a^2}$$

$$\begin{array}{c|ccc} & 1 & -(a+1) & +a \\ a & & +a & -a \\ \hline & 1 & -1 & \boxed{0} \end{array} \Rightarrow x^2-(a+1)x+a=(x-a)(x-1)$$

$$\begin{array}{c|cccc} & 1 & 0 & 0 & -a^3 \\ a & & +a & +a^2 & +a^3 \\ \hline & 1 & +a & +a^2 & \boxed{0} \end{array} \Rightarrow x^3-a^3=(x-a)(x^2+ax+a^2)$$

29) $\displaystyle \lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}=\frac{0}{0}(\mathrm{I})=\lim_{x\to 1}\frac{(\sqrt{x}-1)(\sqrt{x}+1)}{(x-1)(\sqrt{x}+1)}=\lim_{x\to 1}\frac{(\sqrt{x})^2-(1)^2}{(x-1)(\sqrt{x}+1)}=\lim_{x\to 1}\frac{x-1}{(x-1)(\sqrt{x}+1)}=\lim_{x\to 1}\frac{1}{(\sqrt{x}+1)}=\frac{1}{\sqrt{1}+1}=\frac{1}{2}$

30) $\displaystyle \lim_{x\to +\infty}\frac{\sqrt[3]{x^2-2x+3}}{x-5}=\frac{+\infty}{+\infty}(\mathrm{I})=\begin{cases}\displaystyle \lim_{x\to +\infty}\frac{\sqrt[3]{x^2-2x+3}}{x-5}=\lim_{x\to +\infty}\frac{\sqrt[3]{\frac{x^2}{x^3}-\frac{2x}{x^3}+\frac{3}{x^3}}}{\frac{x}{x}-\frac{5}{x}}=\lim_{x\to +\infty}\frac{\sqrt[3]{\frac{1}{x}-\frac{2}{x^2}+\frac{3}{x^3}}}{1-\frac{5}{x}}=\frac{\sqrt[3]{0-0+0}}{1-0}=0\\ \displaystyle \lim_{x\to +\infty}\frac{\sqrt[3]{x^2}}{x}=\lim_{x\to +\infty}\frac{x^{\frac{2}{3}}}{x}=\lim_{x\to +\infty}\frac{1}{x^{\frac{1}{3}}}=\lim_{x\to +\infty}\frac{1}{\sqrt[3]{x}}=\frac{1}{+\infty}=0\end{cases}$

31) $\displaystyle \lim_{x\to -\infty}\frac{\sqrt{3x+x^4}}{2+5x}=\lim_{x\to +\infty}\frac{\sqrt{3(-x)+(-x)^4}}{2+5(-x)}=\lim_{x\to +\infty}\frac{\sqrt{-3x+x^4}}{2-5x}=\frac{+\infty}{-\infty}(\mathrm{I})=$

$\displaystyle =\begin{cases}\displaystyle \lim_{x\to +\infty}\frac{\sqrt{\frac{-3x}{x^4}+\frac{x^4}{x^4}}}{\frac{2}{x^2}-\frac{5x}{x^2}}=\lim_{x\to +\infty}\frac{\sqrt{\frac{-3}{x^3}+1}}{\frac{2}{x^2}-\frac{5}{x}}=\frac{\sqrt{0+1}}{0-0}=\frac{1}{0}=-\infty\\ \displaystyle \lim_{x\to +\infty}\frac{\sqrt{x^4}}{-5x}=\lim_{x\to +\infty}\frac{x^2}{-5x}=\lim_{x\to +\infty}\frac{x}{-5}=\frac{+\infty}{-5}=-\infty\end{cases}$

32) $\displaystyle \lim_{x\to +\infty}\frac{\sqrt{4x+x^2}}{2+5x^2}=\frac{+\infty}{+\infty}(\mathrm{I})=\begin{cases}\displaystyle \lim_{x\to +\infty}\frac{\sqrt{\frac{4x}{x^4}+\frac{x^2}{x^4}}}{\frac{2}{x^2}+\frac{5x^2}{x^2}}=\lim_{x\to +\infty}\frac{\sqrt{\frac{4}{x^3}+\frac{1}{x^2}}}{\frac{2}{x^2}+5}=\frac{\sqrt{0+0}}{0+5}=\frac{0}{5}=0\\ \displaystyle \lim_{x\to +\infty}\frac{\sqrt{x^2}}{5x^2}=\lim_{x\to +\infty}\frac{x}{5x^2}=\lim_{x\to +\infty}\frac{2}{5x}=\frac{2}{+\infty}=0\end{cases}$

33) $\lim_{x\to-\infty}\frac{5x+1}{\sqrt[3]{x^2-3x-1}}=\lim_{x\to+\infty}\frac{5(-x)+1}{\sqrt[3]{(-x)^2-3(-x)-1}}=\lim_{x\to+\infty}\frac{-5x+1}{\sqrt[3]{x^2+3x-1}}=\frac{-\infty}{+\infty}(\text{I})=$

$$=\begin{cases}\lim_{x\to+\infty}\frac{-5x+1}{\sqrt[3]{x^2+3x-1}}=\lim_{x\to+\infty}\frac{\frac{-5x}{x}+\frac{1}{x}}{\sqrt[3]{\frac{x^2}{x^3}+\frac{3x}{x^3}-\frac{1}{x^3}}}=\lim_{x\to+\infty}\frac{-5+\frac{1}{x}}{\sqrt[3]{\frac{1}{x}+\frac{3}{x^2}-\frac{1}{x^3}}}=\frac{-5+0}{\sqrt[3]{0+0-0}}=\frac{-5}{0}=-\infty\\ \lim_{x\to+\infty}\frac{-5x+1}{\sqrt[3]{x^2+3x-1}}=\lim_{x\to+\infty}\frac{-5x}{\sqrt[3]{x^2}}=\lim_{x\to+\infty}\frac{-5x}{x^{\frac{2}{3}}}=\lim_{x\to+\infty}(-5x^{\frac{1}{3}})=\lim_{x\to+\infty}(-5\sqrt[3]{x})=-\infty\end{cases}$$

34) $\lim_{x\to7}\frac{2-\sqrt{x-3}}{x^2-49}=\frac{0}{0}(\text{I})=\lim_{x\to7}\frac{(2-\sqrt{x-3})(2+\sqrt{x-3})}{(x^2-49)(2+\sqrt{x-3})}=\lim_{x\to7}\frac{(2)^2-(\sqrt{x-3})^2}{(x^2-49)(2+\sqrt{x-3})}=$

$=\lim_{x\to7}\frac{4-(x-3)}{(x^2-49)(2+\sqrt{x-3})}=\lim_{x\to7}\frac{4-x+3}{(x^2-49)(2+\sqrt{x-3})}=\lim_{x\to7}\frac{7-x}{(x^2-49)(2+\sqrt{x-3})}=$

$=\lim_{x\to7}\frac{-(x-7)}{(x-7)(x+7)(2+\sqrt{x-3})}=\lim_{x\to7}\frac{-1}{(x+7)(2+\sqrt{x-3})}=\frac{-1}{14\cdot(2+2)}=-\frac{1}{56}$

35) $\lim_{x\to+\infty}\frac{3^x}{10x^2+5x-2}=\frac{+\infty}{+\infty}(\text{I})=+\infty$

Las funciones exponenciales de base mayor que 1 son infinitos de orden superior a cualquier potencia de x.

36) $\lim_{x\to+\infty}\frac{\sqrt{x^5-3}}{10x^2-9}=\frac{+\infty}{+\infty}=\lim_{x\to+\infty}\frac{\sqrt{\frac{x^5}{x^5}-\frac{3}{x^5}}}{\frac{10x^2}{x^{5/2}}-\frac{9}{x^{5/2}}}=\lim_{x\to+\infty}\frac{\sqrt{1-\frac{3}{x^5}}}{\frac{10}{x^{1/2}}-\frac{9}{x^{5/2}}}=\frac{1}{0}=+\infty$

<u>Otra forma</u>

$\lim_{x\to+\infty}\frac{\sqrt{x^5-3}}{10x^2-9}=\frac{+\infty}{+\infty}(\text{I})=\lim_{x\to+\infty}\frac{\sqrt{x^5}}{10x^2}=\lim_{x\to+\infty}\frac{x^{\frac{5}{2}}}{10x^2}=\lim_{x\to+\infty}\frac{x^{\frac{1}{2}}}{10}=\lim_{x\to+\infty}\frac{\sqrt{x}}{10}=+\infty$

37) $\lim_{x\to-\infty}\frac{\sqrt[3]{x^5-3}}{10x^2-9}=\lim_{x\to+\infty}\frac{\sqrt[3]{(-x)^5-3}}{10(-x)^2-9}=\lim_{x\to+\infty}\frac{\sqrt[3]{-x^5-3}}{10x^2-9}=\frac{-\infty}{+\infty}(\text{I})=\lim_{x\to+\infty}\frac{\sqrt[3]{\frac{-x^5}{x^6}-\frac{3}{x^6}}}{\frac{10x^2}{x^2}-\frac{9}{x^2}}=$

$=\lim_{x\to+\infty}\frac{\sqrt[3]{\frac{-1}{x^6}-\frac{3}{x^6}}}{10-\frac{9}{x^2}}=\frac{\sqrt[3]{0-0}}{10-0}=\frac{0}{10}=0$

38) $\lim_{x\to+\infty}\frac{\log(x^3+1)}{10x^2+1}=\frac{+\infty}{+\infty}(\text{I})=0$

Las potencias de x son infinitos de orden superior a cualquier función logarítmica.

39) $\lim_{x\to+\infty}\frac{5^x}{\log(x^3+1)}=\frac{+\infty}{+\infty}(\text{I})=+\infty$

Las funciones exponenciales de base mayor que 1 son infinitos de orden superior a cualquier función logarítmica.

40) $\lim_{x\to+\infty}(2^x-\sqrt{x^5-1})=\infty-\infty(\text{I})=+\infty$

Las funciones exponenciales de base mayor que 1 son infinitos de orden superior a cualquier potencia de *x*.

41) $\lim_{x\to+\infty}(10x^2-\sqrt{x^5-1})=\infty-\infty(\text{I})=-\infty$

Dadas dos potencias de *x* la de mayor exponente es un infinito de orden superior

42) $\lim_{x\to+\infty}[\log(x^3)-10x^2]=\infty-\infty(\text{I})=-\infty$

Las potencias de *x* son infinitos de orden superior a cualquier función logarítmica.

43) $\lim_{x\to+\infty}(e^x-x^3)=\infty-\infty(\text{I})=+\infty$

Las funciones exponenciales de base mayor que 1 son infinitos de orden superior a cualquier potencia de *x*.

44) $\lim_{x\to+\infty}\frac{x^2+1}{e^x}=\frac{+\infty}{+\infty}(\text{I})=0$

Las funciones exponenciales de base mayor que 1 son infinitos de orden superior a cualquier potencia de *x*.

45) $\lim_{x\to+\infty}(\sqrt{x^2+x}-\sqrt{x+7})=\infty-\infty(\text{I})=+\infty$

Dadas dos potencias de *x* la de mayor exponente es un infinito de orden superior

46) $\lim_{x\to+\infty}\frac{\ln(x^2+1)}{x}=\frac{+\infty}{+\infty}=0$

Las potencias de *x* son infinitos de orden superior a cualquier función logarítmica.

47) $\lim_{x\to-\infty}(0{,}5^x+1)=\lim_{x\to+\infty}(0{,}5^{-x}+1)=0{,}5^{-\infty}+1=+\infty+1=+\infty$

48) $\lim_{x\to-\infty}(2^{x+1}-5)=2^{-\infty}-5=0-5=-5$

49) $\lim_{x\to+\infty}\left(1{,}2^x-\frac{3x^2}{x+1}\right)=\infty-\infty(\text{I})=+\infty$

Las funciones exponenciales de base mayor que 1 son infinitos de orden superior a cualquier potencia de *x*.

50) $\lim_{x\leftarrow-\infty}\left(\frac{2x+7}{x}\right)^{1+3x}=\left[\lim_{x\leftarrow-\infty}\left(\frac{2x+7}{x}\right)\right]^{\lim_{x\leftarrow-\infty}(1+3x)}\underset{(*)}{=}2^{-\infty}=0$

$$(*)\lim_{x\to-\infty}\left(\frac{2x+7}{x}\right)=\frac{-\infty}{-\infty}(\text{I})=\lim_{x\to-\infty}\left(\frac{\frac{2x}{x}+\frac{7}{x}}{\frac{x}{x}}\right)=\lim_{x\leftarrow-\infty}\left(\frac{2+\frac{7}{x}}{1}\right)=\frac{2+0}{1}=2$$

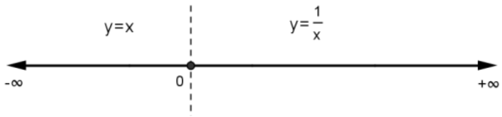
51) $\lim_{x\to+\infty} \frac{x+\log x}{\log x} = \frac{+\infty}{+\infty}(\mathrm{I}) = \lim_{x\to+\infty}\left(\frac{x}{\log x}+\frac{\log x}{\log x}\right) = \lim_{x\to+\infty}\left(\frac{x}{\log x}+1\right) = +\infty$

Las potencias de *x* son infinitos de orden superior a cualquier función logarítmica.

52) $\lim_{x\to+\infty} \frac{3\cdot 2^x}{2^x+1} = \frac{+\infty}{+\infty}(\mathrm{I}) = \lim_{x\to+\infty} \frac{3\cdot 2^x}{2^x} = \lim_{x\to+\infty} 3 = 3$

<u>Ejercicio 5</u>: Calcula los siguientes límites

1) $f(x)=\begin{cases} x & \text{si} \quad x\le 0 \\ \dfrac{1}{x} & \text{si} \quad x>0 \end{cases}$



$$\left.\begin{array}{l} \text{Dom}(y=x)=\Re \quad \Rightarrow \quad (-\infty,0]\in \text{Dom}(f) \\ \text{Dom}\left(y=\dfrac{1}{x}\right)=\Re-\{0\} \quad \Rightarrow \quad (0,+\infty)\in \text{Dom}(f) \end{array}\right\} \Rightarrow \text{Dom}(f)=\Re$$

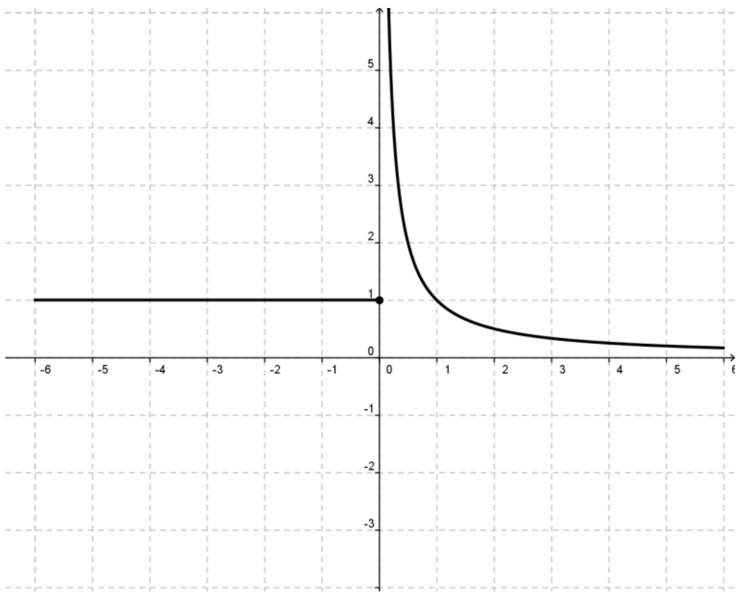
- $\lim\limits_{x\to-\infty} f(x)=\lim\limits_{x\to-\infty} x=-\infty$

- $\lim\limits_{x\to+\infty} f(x)=\lim\limits_{x\to+\infty} \dfrac{1}{x}=\dfrac{1}{+\infty}=0$

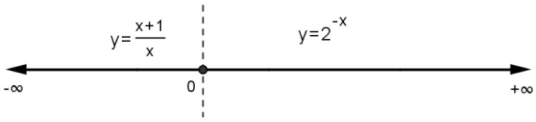
- $\lim\limits_{x\to-3} f(x)=\lim\limits_{x\to-3} x=-3$

- $\lim\limits_{x\to2} f(x)=\lim\limits_{x\to2} \dfrac{1}{x}=\dfrac{1}{2}$

- $\lim\limits_{x\to0} f(x)=\begin{cases} \lim\limits_{x\to0^-} x=0 \\ \lim\limits_{x\to0^+} \dfrac{1}{x}=+\infty \end{cases} \Rightarrow \nexists \lim\limits_{x\to0} f(x)$



2) $f(x)=\begin{cases}\dfrac{x+1}{x} & \text{si} \quad x<0\\ 2^{-x} & \text{si} \quad x\geq 0\end{cases}$



$$\left.\begin{array}{l}\text{Dom}\left(y=\dfrac{x+1}{x}\right)=\Re-\{0\} \quad \Rightarrow \quad (-\infty,0)\in \text{Dom}(f)\\ \text{Dom}\left(y=2^{-x}\right)=\Re \quad \Rightarrow \quad [0,+\infty)\in \text{Dom}(f)\end{array}\right\}\Rightarrow \text{Dom}(f)=\Re$$

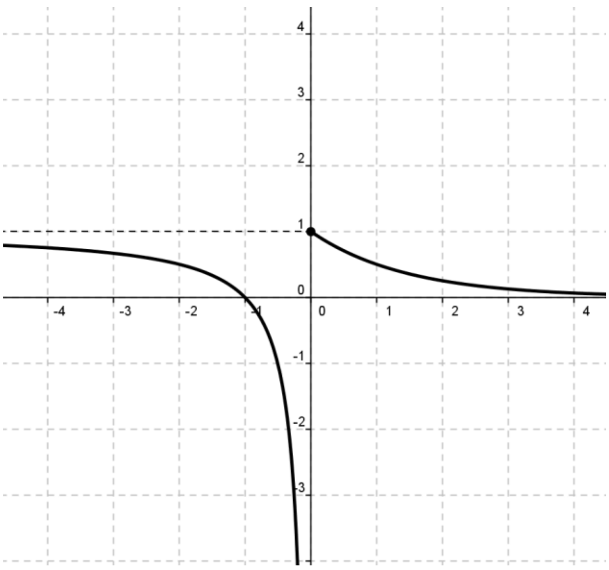
- $\lim\limits_{x\to-\infty} f(x)=\lim\limits_{x\to-\infty}\dfrac{x+1}{x}=\dfrac{-\infty}{-\infty}(\text{I})=\lim\limits_{x\to-\infty}\dfrac{\frac{x}{x}+\frac{1}{x}}{\frac{x}{x}}=\lim\limits_{x\to-\infty}\dfrac{1+\frac{1}{x}}{1}=\dfrac{1}{1}=1 \quad \Rightarrow \quad y=1$ es A.H. por la izquierda

- $\lim\limits_{x\to+\infty} f(x)=\lim\limits_{x\to+\infty} 2^{-x}=2^{-(+\infty)}=2^{-\infty}=0^{+} \quad \Rightarrow \quad y=0$ es A.H. por la derecha

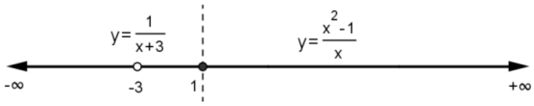
- $\lim\limits_{x\to-3} f(x)=\lim\limits_{x\to-3}\dfrac{x+1}{x}=\dfrac{-3+1}{-3}=\dfrac{-2}{-3}=\dfrac{2}{3}$

- $\lim\limits_{x\to2} f(x)=\lim\limits_{x\to2} 2^{-x}=2^{-2}=\dfrac{1}{4}$

- $\lim\limits_{x\to0} f(x)=\begin{cases}\lim\limits_{x\to0^-}\dfrac{x+1}{x}=\dfrac{1}{0^-}=-\infty\\ \lim\limits_{x\to0^+} 2^{-x}=2^{0}=1\end{cases} \quad \Rightarrow \quad \nexists \lim\limits_{x\to0} f(x) \qquad x=0$ es A.V. por la izquierda



3) $f(x)=\begin{cases}\dfrac{1}{x+3} & \text{si} \quad x<1\\[2mm] \dfrac{x^2-1}{x} & \text{si} \quad x\geq 1\end{cases}$



$$\left.\begin{array}{l}\text{Dom}\left(y=\dfrac{1}{x+3}\right)=\Re-\{-3\} \quad\Rightarrow\quad (-\infty,1)-\{-3\}\in \text{Dom}(f)\\[2mm] \text{Dom}\left(\dfrac{x^2-1}{x}\right)=\Re-\{0\} \quad\Rightarrow\quad [1,+\infty)\in \text{Dom}(f)\end{array}\right\}\Rightarrow \text{Dom}(f)=\Re-\{-3\}$$

- $\lim\limits_{x\to-\infty} f(x)=\lim\limits_{x\to-\infty}\dfrac{1}{x+3}=\dfrac{1}{-\infty}=0^- \quad\Rightarrow\quad y=0$ es A.H. por la izquierda

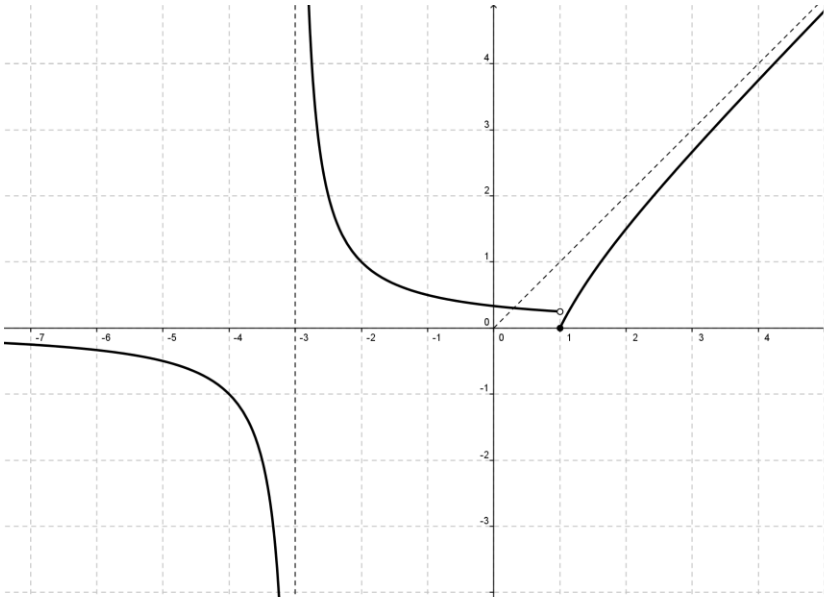
- $\lim\limits_{x\to+\infty} f(x)=\lim\limits_{x\to+\infty}\dfrac{x^2-1}{x}=\dfrac{+\infty}{+\infty}(\text{I})=\lim\limits_{x\to+\infty}\dfrac{\dfrac{x^2}{x^2}-\dfrac{1}{x^2}}{\dfrac{x}{x^2}}=\lim\limits_{x\to+\infty}\dfrac{1-\dfrac{1}{x^2}}{\dfrac{1}{x}}=\dfrac{1-0}{0}=+\infty$

- $\lim\limits_{x\to0} f(x)=\lim\limits_{x\to0}\dfrac{1}{x+3}=\dfrac{1}{3}$

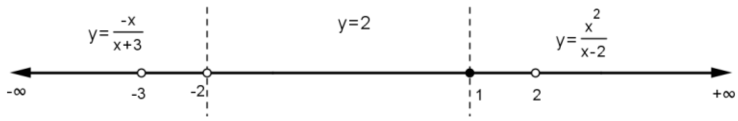
- $\lim\limits_{x\to-3} f(x)=\lim\limits_{x\to-3}\dfrac{1}{x+3}=\dfrac{1}{0}=\begin{cases}\lim\limits_{x\to-3^-}\dfrac{1}{x+3}=\dfrac{1}{0^-}=-\infty\\[2mm] \lim\limits_{x\to-3^+}\dfrac{1}{x+3}=\dfrac{1}{0^+}=+\infty\end{cases} \quad\Rightarrow\quad x=-3$ es A.V.

- $\lim\limits_{x\to2} f(x)=\lim\limits_{x\to2}\dfrac{x^2-1}{x}=\dfrac{2^2-1}{2}=\dfrac{3}{2}$

- $\lim\limits_{x\to1} f(x)=\begin{cases}\lim\limits_{x\to1^-}\dfrac{1}{x+3}=\dfrac{1}{4}\\[2mm] \lim\limits_{x\to1^+}\dfrac{x^2-1}{x}=\dfrac{0}{1}=0\end{cases} \quad\Rightarrow\quad \nexists \lim\limits_{x\to0} f(x)$



4) $f(x)=\begin{cases} \dfrac{-x}{x+3} & \text{si} \quad x<-2 \\ 2 & \text{si} \quad -2<x<1 \\ \dfrac{x^2}{x-2} & \text{si} \quad x\geq 1 \end{cases}$



$$\left.\begin{array}{l} \text{Dom}\left(y=\dfrac{-x}{x+3}\right)=\Re-\{-3\} \quad \Rightarrow \quad (-\infty,-2)-\{-3\}\in \text{Dom}(f) \\ \text{Dom}\left(y=2\right)=\Re \quad \Rightarrow \quad (-2,1)\in \text{Dom}(f) \\ \text{Dom}\left(\dfrac{x^2}{x-2}\right)=\Re-\{2\} \quad \Rightarrow \quad [1,+\infty)-\{2\}\in \text{Dom}(f) \end{array}\right\} \Rightarrow \text{Dom}(f)=\Re-\{-3,-2,2\}$$

- $\lim\limits_{x\to-\infty} f(x)=\lim\limits_{x\to-\infty}\dfrac{-x}{x+3}=\dfrac{+\infty}{-\infty}(\text{I})=\lim\limits_{x\to-\infty}\dfrac{\frac{-x}{x}}{\frac{x}{x}+\frac{3}{x}}=\lim\limits_{x\to-\infty}\dfrac{-1}{1+\frac{3}{x}}=\dfrac{-1}{1+0}=-1 \Rightarrow y=-1$ es A.H. por la izquierda

- $\lim\limits_{x\to+\infty} f(x)=\lim\limits_{x\to+\infty}\dfrac{x^2}{x-2}=\dfrac{+\infty}{+\infty}(\text{I})=\lim\limits_{x\to+\infty}\dfrac{\frac{x^2}{x^2}}{\frac{x}{x^2}-\frac{2}{x^2}}=\lim\limits_{x\to+\infty}\dfrac{1}{\frac{1}{x}-\frac{2}{x^2}}=\dfrac{1}{0}=+\infty$

- $\lim\limits_{x\to-5} f(x)=\lim\limits_{x\to-5}\dfrac{-x}{x+3}=\dfrac{5}{2}$

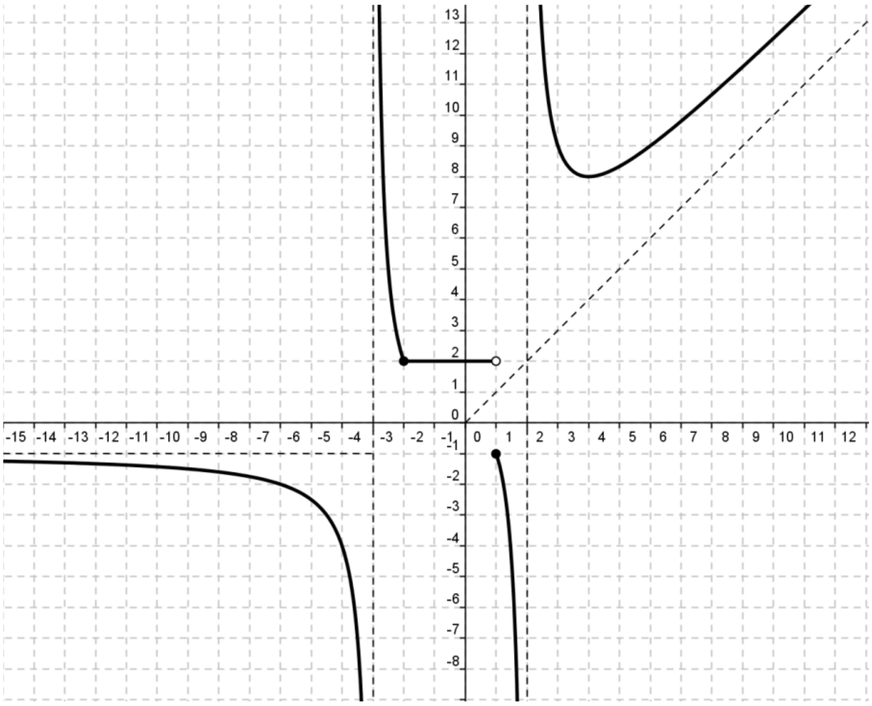
- $\lim\limits_{x\to-3} f(x)=\lim\limits_{x\to-3}\dfrac{-x}{x+3}=\dfrac{3}{0}=\begin{cases}\lim\limits_{x\to-3^-}\dfrac{-x}{x+3}=\dfrac{3}{0^-}=-\infty \\ \lim\limits_{x\to-3^+}\dfrac{-x}{x+3}=\dfrac{3}{0^+}=+\infty\end{cases} \Rightarrow \quad x=-3$ es A.V.

- $\lim_{x\to -2} f(x) = \begin{cases} \lim_{x\to -2^-} \dfrac{-x}{x+3} = \dfrac{2}{1} = 2 \\ \lim_{x\to -2^+} 2 = 2 \end{cases} \Rightarrow \lim_{x\to -2} f(x) = 2$

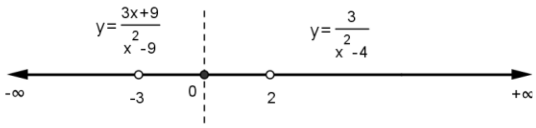
- $\lim_{x\to 0} f(x) = \lim_{x\to 0} 2 = 2$

- $\lim_{x\to 1} f(x) = \begin{cases} \lim_{x\to 1^-} 2 = 2 \\ \lim_{x\to 1^+} \dfrac{x^2}{x-2} = \dfrac{1}{-1} = -1 \end{cases} \Rightarrow \nexists \lim_{x\to 0} f(x)$

- $\lim_{x\to 2} f(x) = \lim_{x\to 2} \dfrac{x^2}{x-2} = \dfrac{4}{0} = \begin{cases} \lim_{x\to 2^-} \dfrac{x^2}{x-2} = \dfrac{4}{0^-} = -\infty \\ \lim_{x\to 2^+} \dfrac{x^2}{x-2} = \dfrac{4}{0^+} = +\infty \end{cases} \Rightarrow x = 2$ es A.V.



5) $f(x)=\begin{cases}\dfrac{3x+9}{x^2-9} & \text{si} \quad x\le 0\\[2mm] \dfrac{3}{x^2-4} & \text{si} \quad x>0\end{cases}$



$$\left.\begin{array}{ll}\text{Dom}\left(y=\dfrac{3x+9}{x^2-9}\right)=\Re-\{\pm 3\} & \Rightarrow \quad (-\infty,0]-\{-3\}\in \text{Dom}(f)\\ \text{Dom}\left(\dfrac{3}{x^2-4}\right)=\Re-\{\pm 2\} & \Rightarrow \quad (0,+\infty)-\{2\}\in \text{Dom}(f)\end{array}\right\}\Rightarrow \text{Dom}(f)=\Re-\{-3,2\}$$

- $\lim\limits_{x\to-\infty} f(x)=\lim\limits_{x\to-\infty}\dfrac{3x+9}{x^2-9}=\dfrac{-\infty}{+\infty}(\text{I})\lim\limits_{x\to-\infty}\dfrac{\frac{3x}{x^2}+\frac{9}{x^2}}{\frac{x^2}{x^2}-\frac{9}{x^2}}=\lim\limits_{x\to-\infty}\dfrac{\frac{3}{x}+\frac{9}{x^2}}{1-\frac{9}{x^2}}=\dfrac{0}{1}=0^- \Rightarrow y=0$ es A.H. por la izquierda

- $\lim\limits_{x\to+\infty} f(x)=\lim\limits_{x\to+\infty}\dfrac{3}{x^2-4}=\dfrac{3}{+\infty}=0^+ \quad \Rightarrow \quad y=0$ es A.H. por la derecha

- $\lim\limits_{x\to-5} f(x)=\lim\limits_{x\to-5}\dfrac{3x+9}{x^2-9}=\dfrac{-15+9}{25-9}=-\dfrac{6}{16}=-\dfrac{3}{8}$

- $\lim\limits_{x\to-3} f(x)=\lim\limits_{x\to-3}\dfrac{3x+9}{x^2-9}=\dfrac{0}{0}(\text{I})=\lim\limits_{x\to-3}\dfrac{3(x+3)}{(x-3)(x+3)}=\lim\limits_{x\to-3}\dfrac{3}{x-3}=\dfrac{3}{-6}=-\dfrac{1}{2} \quad \Rightarrow \quad \lim\limits_{x\to-3} f(x)=-\dfrac{1}{2}$

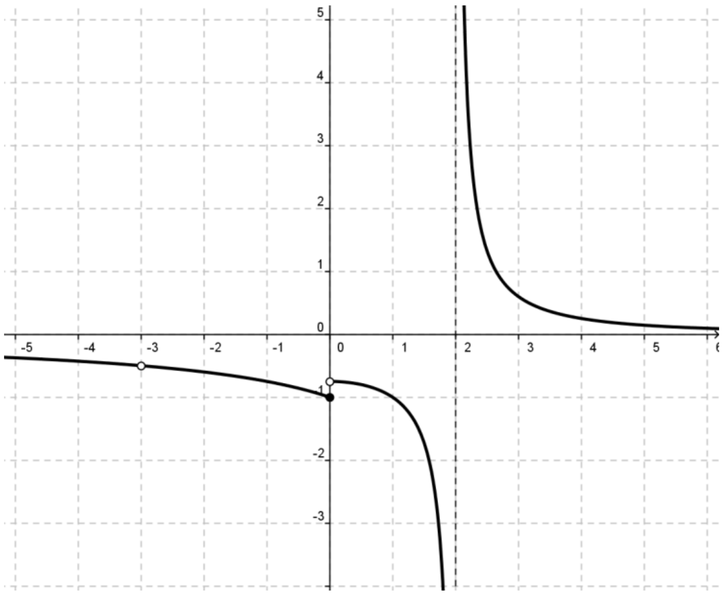
<u>Observación</u>

$$\left.\begin{array}{l}\exists \lim\limits_{x\to-3} f(x)=-\dfrac{1}{2}\\ \nexists\, f(-3)\end{array}\right\}\Rightarrow x=-3 \text{ discontinuidad evitable ("punto en blanco")}$$

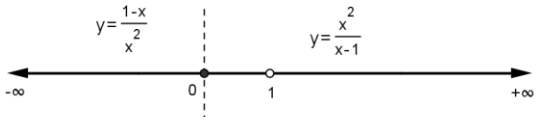
- $\lim\limits_{x\to0} f(x)=\begin{cases}\lim\limits_{x\to0^-}\dfrac{3x+9}{x^2-9}=\dfrac{9}{-9}=-1\\ \lim\limits_{x\to0^+}\dfrac{3}{x^2-4}=-\dfrac{3}{4}\end{cases} \quad \Rightarrow \quad \nexists \lim\limits_{x\to0} f(x)$

- $\lim\limits_{x\to1} f(x)=\lim\limits_{x\to1}\dfrac{3}{x^2-4}=\dfrac{3}{(1)^2-4}=\dfrac{3}{-3}=-1 \quad \Rightarrow \quad \lim\limits_{x\to1} f(x)=-1$

- $\lim\limits_{x\to2} f(x)=\lim\limits_{x\to2}\dfrac{3}{x^2-4}=\dfrac{3}{0}=\begin{cases}\lim\limits_{x\to2^-}\dfrac{3}{x^2-4}=\dfrac{3}{0^-}=-\infty\\ \lim\limits_{x\to2^+}\dfrac{3}{x^2-4}=\dfrac{3}{0^+}=+\infty\end{cases} \quad \Rightarrow \quad x=2$ es A.V.



6) $f(x)=\begin{cases}\dfrac{1-x}{x^2} & \text{si} \quad x<0\\ \dfrac{x^2}{x-1} & \text{si} \quad x\geq 0\end{cases}$



$$\left.\begin{array}{l}\text{Dom}\left(y=\dfrac{1-x}{x^2}\right)=\Re-\{0\} \quad \Rightarrow \quad (-\infty,0)\in\text{Dom}(f)\\ \text{Dom}\left(\dfrac{x^2}{x-1}\right)=\Re-\{1\} \quad \Rightarrow \quad [0,+\infty)-\{1\}\in\text{Dom}(f)\end{array}\right\}\Rightarrow \text{Dom}(f)=\Re-\{1\}$$

- $\lim\limits_{x\to-\infty} f(x)=\lim\limits_{x\to-\infty}\dfrac{1-x}{x^2}=\dfrac{+\infty}{+\infty}(\text{I})=\lim\limits_{x\to-\infty}\dfrac{\dfrac{1}{x^2}-\dfrac{x}{x^2}}{\dfrac{x^2}{x^2}}=\lim\limits_{x\to-\infty}\dfrac{\dfrac{1}{x^2}-\dfrac{1}{x}}{1}=\dfrac{0}{1}=0^+\Rightarrow \quad y=0$ es A.H. por la izquierda

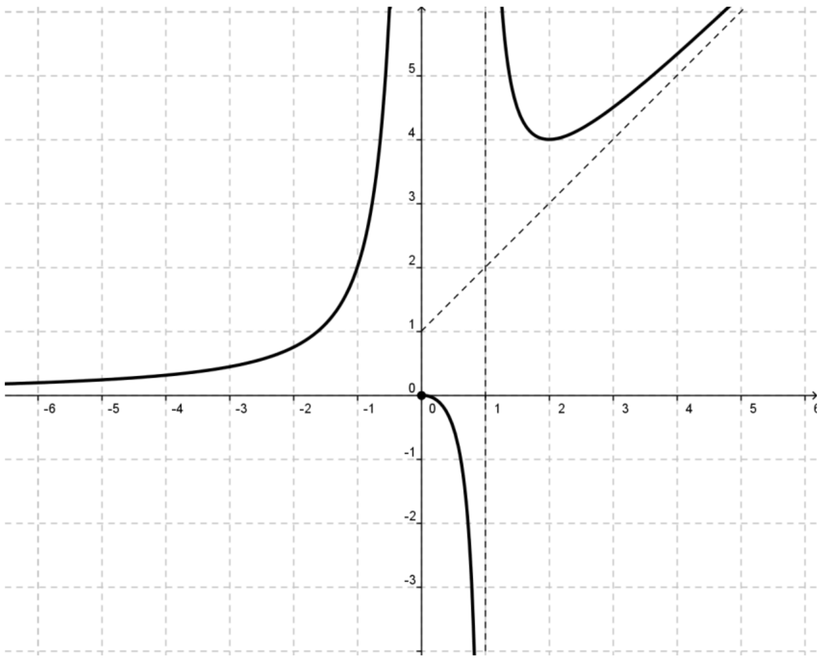
- $\lim\limits_{x\to+\infty} f(x)=\lim\limits_{x\to+\infty}\dfrac{x^2}{x-1}=\dfrac{+\infty}{+\infty}(\text{I})=\lim\limits_{x\to+\infty}\dfrac{\dfrac{x^2}{x^2}}{\dfrac{x}{x^2}-\dfrac{1}{x^2}}=\lim\limits_{x\to+\infty}\dfrac{1}{\dfrac{1}{x}-\dfrac{1}{x^2}}=\dfrac{1}{0}=+\infty$

- $\lim\limits_{x\to-1} f(x)=\lim\limits_{x\to-5}\dfrac{1-x}{x^2}=\dfrac{1-(-1)}{(-1)^2}=\dfrac{2}{1}=2\Rightarrow\lim\limits_{x\to-1} f(x)=2$

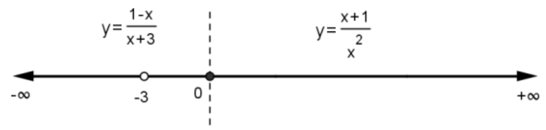
- $\lim\limits_{x\to0} f(x)=\begin{cases}\lim\limits_{x\to0^-}\dfrac{1-x}{x^2}=\dfrac{1}{0^+}=+\infty\\ \lim\limits_{x\to0^+}\dfrac{x^2}{x-1}=\dfrac{0}{-1}=0\end{cases} \quad\Rightarrow\quad \nexists\lim\limits_{x\to0} f(x) \quad x=0$ es A.V. por la izquierda

- $\lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{x^2}{x-1} = \frac{4}{1} = 4 \quad \Rightarrow \quad \lim_{x\to 2} f(x) = 4$

- $\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{x^2}{x-1} = \frac{1}{0} = \begin{cases} \lim_{x\to 1^-} \frac{x^2}{x-1} = \frac{1}{0^-} = -\infty \\ \lim_{x\to 1^+} \frac{x^2}{x-1} = \frac{1}{0^+} = +\infty \end{cases} \quad \Rightarrow \quad x = 1$ es A.V.



7) $f(x) = \begin{cases} \frac{1-x}{x+3} & \text{si} \quad x \le 0 \\ \frac{x+1}{x^2} & \text{si} \quad x > 0 \end{cases}$



$$\left.\begin{array}{l} \text{Dom}\left(y = \frac{1-x}{x+3}\right) = \Re - \{-3\} \quad \Rightarrow \quad (-\infty, 0] - \{-3\} \in \text{Dom}(f) \\ \text{Dom}\left(\frac{x+1}{x^2}\right) = \Re - \{0\} \quad \Rightarrow \quad (0, +\infty) \in \text{Dom}(f) \end{array}\right\} \Rightarrow \text{Dom}(f) = \Re - \{-3\}$$

- $\lim_{x\to -\infty} f(x) = \lim_{x\to -\infty} \frac{1-x}{x+3} = \frac{+\infty}{+\infty}(\text{I}) = \lim_{x\to -\infty} \frac{\frac{1}{x} - \frac{x}{x}}{\frac{x}{x} + \frac{3}{x}} = \lim_{x\to -\infty} \frac{\frac{1}{x} - 1}{1 + \frac{3}{x}} = \frac{-1}{1} = -1 \quad \Rightarrow \quad y = -1$ es A.H. por la izquierda

- $\lim_{x\to +\infty} f(x) = \lim_{x\to +\infty} \frac{x+1}{x^2} = \frac{+\infty}{+\infty}(\text{I}) = \lim_{x\to +\infty} \frac{\frac{x}{x^2} + \frac{1}{x^2}}{\frac{x^2}{x^2}} = \lim_{x\to +\infty} \frac{\frac{1}{x} + \frac{1}{x^2}}{1} = \frac{0}{1} = 0^+ \quad \Rightarrow \quad y = 0$ es A.H. por la derecha

- $\lim_{x\to -1} f(x) = \lim_{x\to -5} \frac{1-x}{x+3} = \frac{1-(-1)}{-1+3} = \frac{2}{2} = 1 \quad \Rightarrow \quad \lim_{x\to -1} f(x) = 1$

- $\lim_{x\to -3} f(x) = \lim_{x\to -3} \frac{1-x}{x+3} = \frac{4}{0} = \begin{cases} \lim_{x\to -3^-} \frac{1-x}{x+3} = \frac{4}{0^-} = -\infty \\ \lim_{x\to -3^+} \frac{1-x}{x+3} = \frac{4}{0^+} = +\infty \end{cases} \quad \Rightarrow \quad \nexists \lim_{x\to -3} f(x)$ $\qquad x = -3$ es A.V.

- $\lim_{x\to 0} f(x) = \begin{cases} \lim_{x\to 0^-} \frac{1-x}{x+3} = \frac{1}{3} \\ \lim_{x\to 0^+} \frac{x+1}{x^2} = \frac{1}{0^+} = +\infty \end{cases} \quad \Rightarrow \quad \nexists \lim_{x\to 0} f(x)$ $\qquad x = 0$ es A.V. por la derecha

- $\lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{x+1}{x^2} = \frac{3}{4} \quad \Rightarrow \quad \lim_{x\to 2} f(x) = \frac{3}{4}$