Ejercicio 2: Calcula los siguientes límites

1) $\lim\limits_{x\to 1}\dfrac{x^2-1}{x-1}=\dfrac{0}{0}(\text{I})=\lim\limits_{x\to 1}\dfrac{(x-1)(x+1)}{(x-1)}=\lim\limits_{x\to 1}(x+1)=2$

2) $\lim\limits_{x\to 1}\dfrac{x^3-1}{x^2-1}=\dfrac{0}{0}(\text{I})=\lim\limits_{x\to 1}\dfrac{(x-1)(x^2+x+1)}{(x-1)(x+1)}=\lim\limits_{x\to 1}\dfrac{x^2+x+1}{x+1}=\dfrac{(1)^2+(1)+1}{(1)+1}=\dfrac{3}{2}$

$$\begin{array}{c|cccc} & 1 & 0 & 0 & -1 \\ 1 & & +1 & +1 & +1 \\ \hline & 1 & +1 & +1 & \boxed{0} \end{array} \Rightarrow x^3-1=(x-1)(x^2+x+1)$$

3) $\lim\limits_{x\to -1}\dfrac{2x^2+5x+3}{x^3+2x^2+2x+1}=\dfrac{0}{0}(\text{I})=\lim\limits_{x\to -1}\dfrac{(x+1)(2x+3)}{(x+1)(x^2+x+1)}=\lim\limits_{x\to -1}\dfrac{(2x+3)}{(x^2+x+1)}=\dfrac{1}{1}=1$

$$\begin{array}{c|ccc} & 2 & 5 & 3 \\ -1 & & -2 & -3 \\ \hline & 2 & +3 & \boxed{0} \end{array} \Rightarrow 2x^2+5x+3=(x+1)(2x+3)$$

$$\begin{array}{c|cccc} & 1 & +2 & +2 & +1 \\ -1 & & -1 & -1 & -1 \\ \hline & 1 & +1 & +1 & \boxed{0} \end{array} \Rightarrow x^3+2x^2+2x+1=(x+1)(x^2+x+1)$$

4) $\lim\limits_{x\to 3}\dfrac{x^2-6x+9}{x-3}=\dfrac{0}{0}(\text{I})=\lim\limits_{x\to 3}\dfrac{(x-3)^2}{(x-3)}=\lim\limits_{x\to 3}(x-3)=0$

5) $\lim\limits_{x\to 2}\dfrac{x^2-5x+6}{(x-2)^2}=\dfrac{0}{0}(\text{I})=\lim\limits_{x\to 2}\dfrac{(x-2)(x-3)}{(x-2)^2}=\lim\limits_{x\to 2}\dfrac{x-3}{x-2}=\dfrac{-1}{0}=\begin{cases}\lim\limits_{x\to 2^-}\dfrac{x-3}{x-2}=\dfrac{-1}{0^-}=+\infty \\ \lim\limits_{x\to 2^+}\dfrac{x-3}{x-2}=\dfrac{-1}{0^+}=-\infty\end{cases}\Rightarrow x=2$ es A.V.

6) $\lim\limits_{x\to 0}\dfrac{2x^3+6x^2-3x}{2x^2+5x}=\dfrac{0}{0}(\text{I})=\lim\limits_{x\to 0}\dfrac{x(2x^2+6x-3)}{x(2x+5)}=\lim\limits_{x\to 0}\dfrac{(2x^2+6x-3)}{(2x+5)}=\dfrac{2\cdot(0)^2+6\cdot(0)-3}{2\cdot(0)+5}=-\dfrac{3}{5}$

$$\lim_{x\to+\infty}\frac{2x^3+6x^2-3x}{2x^2+5x}=\frac{+\infty}{+\infty}=(\text{I})=\lim_{x\to+\infty}\frac{\frac{2x^3}{x^3}+\frac{6x^2}{x^3}-\frac{3x}{x^3}}{\frac{2x^2}{x^3}+\frac{5x}{x^3}}=\lim_{x\to+\infty}\frac{2+\frac{6}{x}-\frac{3}{x^2}}{\frac{2}{x}+\frac{5x}{x^2}}=\frac{2+0-0}{0+0}=+\infty$$

$$\lim_{x\to-\infty}\frac{2x^3+6x^2-3x}{2x^2+5x}=\frac{-\infty}{+\infty}=(\text{I})=\lim_{x\to+\infty}\frac{\frac{2x^3}{x^3}+\frac{6x^2}{x^3}-\frac{3x}{x^3}}{\frac{2x^2}{x^3}+\frac{5x}{x^3}}=\lim_{x\to+\infty}\frac{2+\frac{6}{x}-\frac{3}{x^2}}{\frac{2}{x}+\frac{5x}{x^2}}=\frac{2+0-0}{0+0}=-\infty$$

7) $\lim_{x\to 3^+}\left(\dfrac{4x-2}{x-3}\right)^{\frac{1}{x}}=\left(\dfrac{12}{0^+}\right)^{\frac{1}{3}}=(+\infty)^{\frac{1}{3}}=+\infty$

8) $\lim_{x\to +\infty}\dfrac{1+2x-3x^3}{x^2-x^3-4}=\dfrac{-\infty}{-\infty}(\mathrm{I})=\lim_{x\to +\infty}\dfrac{\frac{1}{x^3}+\frac{2x}{x^3}-\frac{3x^3}{x^3}}{\frac{x^2}{x^3}-\frac{x^3}{x^3}-\frac{4}{x^3}}=\lim_{x\to +\infty}\dfrac{\frac{1}{x^3}+\frac{2}{x^2}-3}{\frac{1}{x}-1-\frac{4}{x^3}}=\dfrac{0+0-3}{0-1-0}=\dfrac{-3}{-1}=3$

9) $\lim_{x\to -\infty}\dfrac{5x^2-3x+1}{x^2-x^3-4}=\dfrac{+\infty}{+\infty}(\mathrm{I})=\lim_{x\to -\infty}\dfrac{\frac{5x^2}{x^3}-\frac{3x}{x^3}+\frac{1}{x^3}}{\frac{x^2}{x^3}-\frac{x^3}{x^3}-\frac{4}{x^3}}=\lim_{x\to -\infty}\dfrac{\frac{5}{x}-\frac{3}{x^2}+\frac{1}{x^3}}{\frac{1}{x}-1-\frac{4}{x^3}}=\dfrac{0+0+0}{0-1-0}=\dfrac{0}{-1}=0$

10) $\lim_{x\to -\infty}\dfrac{x^4-3x+1}{2x-3x^3-4}=\dfrac{+\infty}{+\infty}=(\mathrm{I})=\lim_{x\to -\infty}\dfrac{\frac{x^4}{x^4}-\frac{3x}{x^4}+\frac{1}{x^4}}{\frac{2x}{x^4}-\frac{3x^3}{x^4}-\frac{4}{x^4}}=\lim_{x\to -\infty}\dfrac{1-\frac{3}{x^3}+\frac{1}{x^4}}{\frac{2}{x^3}-\frac{3}{x}-\frac{4}{x^4}}=\dfrac{1-0+0}{0-0-0}=\dfrac{1}{0}=+\infty$

11) $\lim_{x\to -\infty}\dfrac{-2x^4-3x+1}{x^3-3x^4-4}=\dfrac{-\infty}{-\infty}=(\mathrm{I})=\lim_{x\to -\infty}\dfrac{\frac{-2x^4}{x^4}-\frac{3x}{x^4}+\frac{1}{x^4}}{\frac{x^3}{x^4}-\frac{3x^4}{x^4}-\frac{4}{x^4}}=\lim_{x\to -\infty}\dfrac{-2-\frac{3}{x^3}+\frac{1}{x^4}}{\frac{1}{x}-3-\frac{4}{x^4}}=\dfrac{-2-0+0}{0-3-0}=\dfrac{2}{3}$

12) $\lim_{x\to 1}\dfrac{2x^3-4x^2+2x}{x^3-x^2-x+1}=\dfrac{0}{0}(\mathrm{I})=\lim_{x\to 1}\dfrac{(x-1)^2\cdot 2x}{(x-1)^2\cdot(x+1)}=\lim_{x\to 1}\dfrac{2x}{(x+1)}=\dfrac{2}{2}=1$

	2	−4	+2	0
+1		+2	−2	0
	2	−2	0	0
+1		+2	0	
	2	0	0	

$\Rightarrow 2x^3-4x^2+2x=(x-1)^2\cdot 2x$

	1	−1	−1	+1
+1		+1	0	0
	1	0	−1	0
+1		+1	+1	
	1	+1	0	

$\Rightarrow x^3-x^2-x+1=(x-1)^2\cdot(x+1)$

13) $\lim\limits_{x\to -2}\dfrac{2x^2+7x+6}{x^3+3x^2+3x+2}=\dfrac{0}{0}(\mathrm{I})=\lim\limits_{x\to -2}\dfrac{(x+2)(2x+3)}{(x+2)(x^2+x+1)}=\lim\limits_{x\to -2}\dfrac{2x+3}{x^2+x+1}=\dfrac{2\cdot(-2)+3}{(-2)^2+(-2)+1}=-\dfrac{1}{3}$

	2	+7	+6
−2		−3	−6
	2	+3	0

$\Rightarrow 2x^2+7x+6=(x+2)(2x+3)$

	1	+3	+3	+2
−2		−2	−2	−2
	1	+1	+1	0

$\Rightarrow x^3+3x^2+3x+2=(x+2)(x^2+x+1)$

14) $\lim\limits_{x\to 1}\dfrac{x^4-x^3+x^2-2x+1}{x^3-x^2+x-1}=\dfrac{0}{0}(\mathrm{I})=\lim\limits_{x\to 1}\dfrac{(x-1)(x^3+x-1)}{(x-1)(x^2+1)}=\lim\limits_{x\to 1}\dfrac{x^3+x-1}{x^2+1}=\dfrac{1}{2}$

	1	−1	+1	−2	+1
1		+1	0	+1	−1
	1	0	+1	−1	0

$\Rightarrow x^4-x^3+x^2-2x+1=(x-1)(x^3+x-1)$

	1	−1	+1	−1
1		+1	0	+1
	1	0	+1	0

$\Rightarrow x^3-x^2+x-1=(x-1)(x^2+1)$

15) $\lim\limits_{x\to \frac{1}{2}}\dfrac{8x^3+4x^2-10x+3}{4x^4-15x^2+13x-3}=\dfrac{0}{0}(\mathrm{I})=\lim\limits_{x\to \frac{1}{2}}\dfrac{\left(x-\frac{1}{2}\right)^2(8x+12)}{\left(x-\frac{1}{2}\right)^2(4x^2+4x-12)}=\lim\limits_{x\to \frac{1}{2}}\dfrac{(8x+12)}{(4x^2+4x-12)}=\dfrac{4+12}{1+2-12}=-\dfrac{16}{9}$

	8	+4	−10	+3
$\frac{1}{2}$		+4	+4	−3
	8	+8	−6	0
$\frac{1}{2}$		+4	+6	
	8	+12	0	

$\Rightarrow\ 8x^3+4x^2-10x+3=\left(x-\dfrac{1}{2}\right)^2\cdot(8x+12)$

	4	0	−15	+13	−3
$\frac{1}{2}$		+2	+1	−7	+3
	4	+2	−14	+6	0
$\frac{1}{2}$		+2	+2	−6	
	4	+4	−12	0	

$\Rightarrow\ 4x^4-15x^2+13x-3=\left(x-\dfrac{1}{2}\right)^2\cdot(4x^2+4x-12)$

16) $\lim_{x\to a}\dfrac{x^6-a^6}{x^3-a^3}=\dfrac{0}{0}(\text{I})=\lim_{x\to a}\dfrac{(x^3-a^3)(x^3+a^3)}{(x^3-a^3)}=\lim_{x\to a}(x^3+a^3)=a^3+a^3=2a^3$

17) $\lim_{x\to a}\dfrac{x^2-(a+1)x+a}{x^2-a^2}=\lim_{x\to a}\dfrac{(x-a)(x-1)}{(x-a)(x+a)}=\lim_{x\to a}\dfrac{x-1}{x+a}=\dfrac{a-1}{2a}$

	1	$-(a+1)$	$+a$
a		$+a$	$-a$
	1	-1	0

$\Rightarrow x^2-(a+1)x+a=(x-a)(x-1)$

18) $\lim_{x\to a}\dfrac{x^3-ax^2-a^2x+a^3}{x^3-3ax^2+3a^2x-a^3}=\dfrac{0}{0}(\text{I})=\lim_{x\to a}\dfrac{(x-a)^2(x+a)}{(x-a)^3}=\lim_{x\to a}\dfrac{x+a}{x-a}=$

- Si $a>0\Rightarrow\begin{cases}\lim_{x\to a^-}\dfrac{x+a}{x-a}=\dfrac{2a}{0^-}=-\infty\\ \lim_{x\to a^+}\dfrac{x+a}{x-a}=\dfrac{2a}{0^+}=+\infty\end{cases}$
- Si $a<0\Rightarrow\begin{cases}\lim_{x\to a^-}\dfrac{x+a}{x-a}=\dfrac{2a}{0^-}=+\infty\\ \lim_{x\to a^+}\dfrac{x+a}{x-a}=\dfrac{2a}{0^+}=-\infty\end{cases}$
- Si $a=0\Rightarrow\lim_{x\to a}\dfrac{x}{x}=\lim_{x\to a}1=1$

	1	$-a$	$-a^2$	$+a^3$
a		$+a$	0	$-a^3$
	1	0	$-a^2$	0
a		$+a$	$+a^2$	
	1	$+a$	0	

$\Rightarrow x^3-ax^2-a^2x+a^3=(x-a)^2\cdot(x+a)$

	1	$-3a$	$+3a^2$	$-a^3$
a		$+a$	$-2a^2$	$+a^3$
	1	$-2a$	$+a^2$	0
a		$+a$	$-a^2$	
	1	$-a$	0	

$\Rightarrow x^3-3ax^2+3a^2x-a^3=(x-a)^2\cdot(x-a)=(x-a)^3$

19) $\lim_{x\to 2}\left(\frac{3}{x^2-5x+6}-\frac{4}{x-2}\right)=\infty-\infty\,(\mathrm{I})=\lim_{x\to 2}\left(\frac{3}{(x-2)(x-3)}-\frac{4}{x-2}\right)=\lim_{x\to 2}\left(\frac{3-4(x-3)}{(x-2)(x-3)}\right)=$

$$=\lim_{x\to 2}\left(\frac{3-4x+12)}{(x-2)(x-3)}\right)=\lim_{x\to 2}\frac{15-4x}{(x-2)(x-3)}=\frac{7}{0}=\begin{cases}\lim_{x\to 2^-}\frac{15-4x}{(x-2)(x-3)}=\frac{7}{(0^-)(-1)}=\frac{7}{0^+}=+\infty\\ \lim_{x\to 2^+}\frac{15-4x}{(x-2)(x-3)}=\frac{7}{(0^+)(-1)}=\frac{7}{0^-}=-\infty\end{cases}$$

20) $\lim_{x\to+\infty}\left(\frac{4x^2-x+3}{3x^2+x-3}\right)^{\frac{x}{1-x}}=\left[\lim_{x\to+\infty}\left(\frac{4x^2-x+3}{3x^2+x-3}\right)\right]^{\lim_{x\to+\infty}\left(\frac{x}{1-x}\right)}=\left(\frac{4}{3}\right)^{-1}=\frac{3}{4}$

- $\lim_{x\to+\infty}\left(\frac{4x^2-x+3}{3x^2+x-3}\right)=\frac{+\infty}{+\infty}(\mathrm{I})=\lim_{x\to+\infty}\frac{\frac{4x^2}{x^2}-\frac{x}{x^2}+\frac{3}{x^2}}{\frac{3x^2}{x^2}+\frac{x}{x^2}-\frac{3}{x^2}}=\lim_{x\to+\infty}\frac{4-\frac{1}{x}+\frac{3}{x^2}}{3+\frac{1}{x}-\frac{3}{x^2}}=\frac{4-0+0}{3+0-0}=\frac{4}{3}$
- $\lim_{x\to+\infty}\left(\frac{x}{1-x}\right)=\frac{+\infty}{-\infty}(\mathrm{I})=\lim_{x\to+\infty}\frac{\frac{x}{x}}{\frac{1}{x}-\frac{x}{x}}=\lim_{x\to+\infty}\frac{1}{\frac{1}{x}-1}=\frac{1}{0-1}=-1$

21) $\lim_{x\to+\infty}\left(\frac{x}{3x^2+2}\right)^{\frac{x^2}{1+x}}=\left[\lim_{x\to+\infty}\left(\frac{x}{3x^2+2}\right)\right]^{\lim_{x\to+\infty}\left(\frac{x^2}{1+x}\right)}=0^{+\infty}=0$

- $\lim_{x\to+\infty}\left(\frac{x}{3x^2+2}\right)=\frac{+\infty}{+\infty}(\mathrm{I})=\lim_{x\to+\infty}\frac{\frac{x}{x^2}}{\frac{3x^2}{x^2}+\frac{2}{x^2}}=\lim_{x\to+\infty}\frac{\frac{1}{x}}{3+\frac{2}{x^2}}=\frac{0}{3+0}=\frac{0}{3}=0$
- $\lim_{x\to+\infty}\left(\frac{x^2}{1+x}\right)=\frac{+\infty}{+\infty}(\mathrm{I})=\lim_{x\to+\infty}\frac{\frac{x^2}{x^2}}{\frac{1}{x^2}+\frac{x}{x^2}}=\lim_{x\to+\infty}\frac{1}{\frac{1}{x^2}+\frac{1}{x}}=\frac{1}{0}=+\infty$

22) $\lim_{x\to+\infty}\left(\frac{x^2+3}{3x^2-5}\right)^{\frac{3x^2}{x-2}}=\left[\lim_{x\to+\infty}\left(\frac{x^2+3}{3x^2-5}\right)\right]^{\lim_{x\to+\infty}\left(\frac{3x^2}{x-2}\right)}=\left(\frac{1}{3}\right)^{+\infty}=0$

- $\lim_{x\to+\infty}\left(\frac{x^2+3}{3x^2-5}\right)=\frac{+\infty}{+\infty}(\mathrm{I})=\lim_{x\to+\infty}\frac{\frac{x^2}{x^2}+\frac{3}{x^2}}{\frac{3x^2}{x^2}-\frac{5}{x^2}}=\lim_{x\to+\infty}\frac{1+\frac{3}{x^2}}{3-\frac{5}{x^2}}=\frac{1+0}{3-0}=\frac{1}{3}$
- $\lim_{x\to+\infty}\left(\frac{3x^2}{x-2}\right)=\frac{+\infty}{+\infty}(\mathrm{I})=\lim_{x\to+\infty}\frac{\frac{3x^2}{x^2}}{\frac{x}{x^2}-\frac{2}{x^2}}=\lim_{x\to+\infty}\frac{3}{\frac{1}{x}-\frac{2}{x^2}}=\frac{3}{0}=+\infty$

23) $\lim_{x\to+\infty}\left(\frac{x^2+3}{3x^2-5}\right)^{\frac{x^2}{2-x}}=\left[\lim_{x\to+\infty}\left(\frac{x^2+3}{3x^2-5}\right)\right]^{\lim_{x\to+\infty}\left(\frac{x^2}{2-x}\right)}=\left(\frac{1}{3}\right)^{-\infty}=3^{+\infty}=+\infty$

- $\lim_{x\to+\infty}\left(\frac{x^2+3}{3x^2-5}\right)=\frac{+\infty}{+\infty}(\mathrm{I})=\lim_{x\to+\infty}\frac{\frac{x^2}{x^2}+\frac{3}{x^2}}{\frac{3x^2}{x^2}-\frac{5}{x^2}}=\lim_{x\to+\infty}\frac{1+\frac{3}{x^2}}{3-\frac{5}{x^2}}=\frac{1+0}{3-0}=\frac{1}{3}$
- $\lim_{x\to+\infty}\left(\frac{3x^2}{2-x}\right)=\frac{+\infty}{-\infty}(\mathrm{I})=\lim_{x\to+\infty}\frac{\frac{3x^2}{x^2}}{\frac{2}{x^2}-\frac{x}{x^2}}=\lim_{x\to+\infty}\frac{3}{\frac{2}{x^2}-\frac{1}{x}}=\frac{3}{0}=-\infty$

24) $\lim_{x\to+\infty}\left(\frac{x^2+3}{3x^2-5}\right)^{\frac{x}{x-2}}=\left[\lim_{x\to+\infty}\left(\frac{x^2+3}{3x^2-5}\right)\right]^{\lim_{x\to+\infty}\left(\frac{x}{x-2}\right)}=\left(\frac{1}{3}\right)^{1}=\frac{1}{3}$

- $\lim_{x\to+\infty}\left(\frac{x^2+3}{3x^2-5}\right)=\frac{+\infty}{+\infty}(\mathrm{I})=\lim_{x\to+\infty}\frac{\frac{x^2}{x^2}+\frac{3}{x^2}}{\frac{3x^2}{x^2}-\frac{5}{x^2}}=\lim_{x\to+\infty}\frac{1+\frac{3}{x^2}}{3-\frac{5}{x^2}}=\frac{1+0}{3-0}=\frac{1}{3}$
- $\lim_{x\to+\infty}\left(\frac{x}{x-2}\right)=\frac{+\infty}{+\infty}(\mathrm{I})=\lim_{x\to+\infty}\frac{\frac{x}{x}}{\frac{x}{x}-\frac{2}{x}}=\lim_{x\to+\infty}\frac{1}{1-\frac{2}{x}}=\frac{1}{1-0}=1$

25) $\lim_{x\to1}\frac{x-1}{\sqrt{x}-1}=\frac{0}{0}(\mathrm{I})=\lim_{x\to1}\frac{(x-1)(\sqrt{x}+1)}{(\sqrt{x}-1)(\sqrt{x}+1)}=\lim_{x\to1}\frac{(x-1)(\sqrt{x}+1)}{(\sqrt{x})^2-1^2}=\lim_{x\to1}\frac{(x-1)(\sqrt{x}+1)}{(x-1)}=\lim_{x\to1}(\sqrt{x}+1)=1+1=2$

26) $\lim_{x\to0}\frac{2-\sqrt{4-x}}{x}=\frac{0}{0}(\mathrm{I})=\lim_{x\to0}\frac{(2-\sqrt{4-x})(2+\sqrt{4-x})}{x(2+\sqrt{4-x})}=\lim_{x\to0}\frac{2^2-(\sqrt{4-x})^2}{x(2+\sqrt{4-x})}=\lim_{x\to0}\frac{4-(4-x)}{x(2+\sqrt{4-x})}=$

$=\lim_{x\to0}\frac{4-4+x}{x(2+\sqrt{4-x})}=\lim_{x\to0}\frac{x}{x(2+\sqrt{4-x})}\underset{\text{simplifica r}}{=}=\lim_{x\to0}\frac{1}{(2+\sqrt{4-x})}=\frac{1}{2+\sqrt{4}}=\frac{1}{2+2}=\frac{1}{4}$

27) $\lim_{x\to+\infty}(\sqrt{x^3+x+1}-x)=\infty-\infty(\mathrm{I})=\lim_{x\to+\infty}(\sqrt{x^3}-x)=\lim_{x\to+\infty}(x^{\frac{3}{2}}-x^1)=+\infty$

28) $\lim_{x\to+\infty}(x-\sqrt{4x^3+x+1})=\infty-\infty(\mathrm{I})=\lim_{x\to+\infty}(x-\sqrt{4x^3})=\lim_{x\to+\infty}(x-2x^{\frac{3}{2}})=-\infty$

29) $\lim_{x\to+\infty}(\sqrt{4x^2+x+1}-5x)=\infty-\infty(\mathrm{I})=\lim_{x\to+\infty}(\sqrt{4x^2}-5x)=\lim_{x\to+\infty}(2x-5x)=-\infty$

30) $\lim_{x\to+\infty}(\sqrt{x^2+x+1}-x)=\infty-\infty(\mathrm{I})=\lim_{x\to+\infty}\frac{(\sqrt{x^2+x+1}-x)(\sqrt{x^2+x+1}+x)}{(\sqrt{x^2+x+1}+x)}=$

$=\lim_{x\to+\infty}\frac{(\sqrt{x^2+x+1})^2-x^2}{(\sqrt{x^2+x+1}+x)}=\lim_{x\to+\infty}\frac{x^2+x+1-x^2}{(\sqrt{x^2+x+1}+x)}=\lim_{x\to+\infty}\frac{x+1}{\sqrt{x^2+x+1}+x}=\frac{+\infty}{+\infty}(\mathrm{I})=\lim_{x\to+\infty}\frac{\frac{x}{x}+\frac{1}{x}}{\sqrt{\frac{x^2}{x^2}+\frac{x}{x^2}+\frac{1}{x^2}}+\frac{x}{x}}=$

$=\lim_{x\to+\infty}\frac{1+\frac{1}{x}}{\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+1}=\frac{1}{\sqrt{1+0+0}+1}=\frac{1}{\sqrt{1}+1}=\frac{1}{2}$

31) $\lim_{x\to+\infty}(\sqrt{x+2}-\sqrt{x-2})=\infty-\infty(\mathrm{I})=\lim_{x\to+\infty}\frac{(\sqrt{x+2}-\sqrt{x-2})(\sqrt{x+2}+\sqrt{x-2})}{(\sqrt{x+2}+\sqrt{x-2})}=$

$=\lim_{x\to+\infty}\frac{(\sqrt{x+2})^2-(\sqrt{x-2})^2}{(\sqrt{x+2}+\sqrt{x-2})}=\lim_{x\to+\infty}\frac{x+2-(x-2)}{(\sqrt{x+2}+\sqrt{x-2})}=\lim_{x\to+\infty}\frac{x+2-x+2}{(\sqrt{x+2}+\sqrt{x-2})}=\lim_{x\to+\infty}\frac{4}{(\sqrt{x+2}+\sqrt{x-2})}=$

$=\frac{4}{\infty+\infty}=\frac{4}{+\infty}=0$

32) $\lim_{x\to+\infty}(x-\sqrt{x^2+x+1})=\infty-\infty(\mathrm{I})=\lim_{x\to+\infty}\frac{(x-\sqrt{x^2+x+1})(x+\sqrt{x^2+x+1})}{(x+\sqrt{x^2+x+1})}=$

$=\lim_{x\to+\infty}\frac{(x)^2-(\sqrt{x^2+x+1})^2}{(x+\sqrt{x^2+x+1})}=\lim_{x\to+\infty}\frac{x^2-(x^2+x+1)^2}{(x+\sqrt{x^2+x+1})}=\lim_{x\to+\infty}\frac{x^2-x^2-x-1}{(x+\sqrt{x^2+x+1})}=\lim_{x\to+\infty}\frac{-x-1}{(x+\sqrt{x^2+x+1})}=$

$=\frac{-\infty}{+\infty}(\mathrm{I})=\lim_{x\to+\infty}\frac{-\frac{x}{x}-\frac{1}{x}}{\frac{x}{x}+\sqrt{\frac{x^2}{x^2}+\frac{x}{x^2}+\frac{1}{x}}}=\lim_{x\to+\infty}\frac{-1-\frac{1}{x}}{1+\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}}=\frac{-1+0}{1+\sqrt{1+0+0}}=\frac{-1}{1+\sqrt{1}}=-\frac{1}{2}$

33) $\lim_{x\to0}\frac{x}{\sqrt{1+x}-\sqrt{1-x}}=\frac{0}{0}(\mathrm{I})=\lim_{x\to0}\frac{x(\sqrt{1+x}+\sqrt{1-x})}{(\sqrt{1+x}-\sqrt{1-x})(\sqrt{1+x}+\sqrt{1-x})}=\lim_{x\to0}\frac{x(\sqrt{1+x}+\sqrt{1-x})}{(\sqrt{1+x})^2-(\sqrt{1-x})^2}=$

$=\lim_{x\to0}\frac{x(\sqrt{1+x}+\sqrt{1-x})}{(1+x)-(1-x)}=\lim_{x\to0}\frac{x(\sqrt{1+x}+\sqrt{1-x})}{1+x-1+x}=\lim_{x\to0}\frac{x(\sqrt{1+x}+\sqrt{1-x})}{2x}\underset{\text{simplifica r}}{=}\lim_{x\to0}\frac{\sqrt{1+x}+\sqrt{1-x}}{2}=$

$=\frac{\sqrt{1+0}+\sqrt{1-0}}{2}=\frac{2}{2}=1$

34) $\lim\limits_{x\to 3}\dfrac{\sqrt[3]{x^3-27}}{\sqrt[3]{x^2+6x-27}}=\lim\limits_{x\to 3}\sqrt[3]{\dfrac{x^3-27}{x^2+6x-27}}=\sqrt[3]{\lim\limits_{x\to 3}\dfrac{x^3-27}{x^2+6x-27}}=\sqrt[3]{\dfrac{9}{4}}$

$\lim\limits_{x\to 3}\dfrac{x^3-27}{x^2+6x-27}=\dfrac{0}{0}(\mathrm{I})=\lim\limits_{x\to 3}\dfrac{(x-3)(x^2+3x+9)}{(x-3)(x+9)}=\lim\limits_{x\to 3}\dfrac{x^2+3x+9}{x+9}=\dfrac{(3)^2+3(3)+9}{3+9}=\dfrac{9+9+9}{12}=\dfrac{27}{12}=\dfrac{9}{4}$

	1	0	0	−27
3		+3	+9	+27
	1	+3	+9	0

$\Rightarrow x^3-27=(x-3)(x^2+3x+9)$

	1	+6	−27
3		+3	+27
	1	+9	0

$\Rightarrow x^2+6x-27=(x-3)(x+9)$

35) $\lim\limits_{x\to 5}\dfrac{3-\sqrt{4+x}}{x-5}=\dfrac{0}{0}(\mathrm{I})=\lim\limits_{x\to 5}\dfrac{(3-\sqrt{4+x})(3+\sqrt{4+x})}{(x-5)(3+\sqrt{4+x})}=\lim\limits_{x\to 5}\dfrac{(3)^2-(\sqrt{4+x})^2}{(x-5)(3+\sqrt{4+x})}=\lim\limits_{x\to 5}\dfrac{9-(4+x)}{(x-5)(3+\sqrt{4+x})}=$

$=\lim\limits_{x\to 5}\dfrac{9-4-x}{(x-5)(3+\sqrt{4+x})}=\lim\limits_{x\to 5}\dfrac{5-x}{(x-5)(3+\sqrt{4+x})}=\lim\limits_{x\to 5}\dfrac{-(x-5)}{(x-5)(3+\sqrt{4+x})}\underset{\text{simplificar}}{=}$

$=\lim\limits_{x\to 5}\dfrac{-1}{(3+\sqrt{4+x})}=\dfrac{-1}{3+\sqrt{4+5}}=\dfrac{-1}{3+\sqrt{9}}=-\dfrac{1}{6}$

36) $\lim\limits_{x\to 4}\dfrac{3-\sqrt{5+x}}{1-\sqrt{5-x}}=\dfrac{0}{0}(\mathrm{I})=\lim\limits_{x\to 4}\dfrac{[(3-\sqrt{5+x})(3+\sqrt{5+x})](1+\sqrt{5-x})}{[(1-\sqrt{5-x})(1+\sqrt{5-x})](3+\sqrt{5+x})}=$

$=\lim\limits_{x\to 4}\dfrac{[(3)^2-(\sqrt{5+x})^2](1+\sqrt{5-x})}{[(1)^2-(\sqrt{5-x})^2](3+\sqrt{5+x})}=\lim\limits_{x\to 4}\dfrac{[9-(5+x)](1+\sqrt{5-x})}{[1-(5-x)](3+\sqrt{5+x})}=\lim\limits_{x\to 4}\dfrac{(9-5-x)(1+\sqrt{5-x})}{(1-5+x)(3+\sqrt{5+x})}=$

$=\lim\limits_{x\to 4}\dfrac{(4-x)(1+\sqrt{5-x})}{(x-4)(3+\sqrt{5+x})}=\lim\limits_{x\to 4}\dfrac{-(x-4)(1+\sqrt{5-x})}{(x-4)(3+\sqrt{5+x})}\underset{\text{simplificar}}{=}\lim\limits_{x\to 4}\dfrac{-(1+\sqrt{5-x})}{(3+\sqrt{5+x})}=\dfrac{-(1+\sqrt{5-4})}{(3+\sqrt{5+4})}=-\dfrac{2}{6}=-\dfrac{1}{3}$

37) $\lim\limits_{x\to 2}\dfrac{x-\sqrt{x+2}}{\sqrt{4x+1}-3}=\dfrac{0}{0}(\mathrm{I})=\lim\limits_{x\to 2}\dfrac{[(x-\sqrt{x+2})(x+\sqrt{x+2})](\sqrt{4x+1}+3)}{[(\sqrt{4x+1}-3)(\sqrt{4x+1}+3)](x+\sqrt{x+2})}=$

$=\lim\limits_{x\to 2}\dfrac{[(x)^2-(\sqrt{x+2})^2](\sqrt{4x+1}+3)}{[(\sqrt{4x+1})^2-(3)^2](x+\sqrt{x+2})}=\lim\limits_{x\to 2}\dfrac{[x^2-(x+2)](\sqrt{4x+1}+3)}{[4x+1-9](x+\sqrt{x+2})}=$

$=\lim\limits_{x\to 2}\dfrac{(x^2-x-2)(\sqrt{4x+1}+3)}{(4x-8)(x+\sqrt{x+2})}\underset{\substack{\text{factorizar}\\ \text{polinomios}}}{=}=\lim\limits_{x\to 2}\dfrac{(x-2)(x+1)((\sqrt{4x+1}+3)}{4(x-2)(x\sqrt{x+2})}\underset{\text{simplificar}}{=}$

$=\lim\limits_{x\to 2}\dfrac{(x+1)(\sqrt{4x+1}+3)}{4(x+\sqrt{x+2})}=\dfrac{(2+1)(\sqrt{4\cdot 2+1}+3)}{4(2+\sqrt{2+2})}=\dfrac{3\cdot(\sqrt{9}+3)}{4\cdot(2+\sqrt{4})}=\dfrac{3\cdot 6}{4\cdot 4}=\dfrac{3\cdot 3\cdot 2}{4\cdot 2\cdot 2}=\dfrac{9}{8}$

38) $$\lim_{\substack{x\to 4\\ x\to 2}} \frac{\sqrt{3x+4}-x}{x-\sqrt{5x-4}} = \frac{0}{0}(\text{I}) = \lim_{x\to 4}\frac{[(\sqrt{3x+4}-x)(\sqrt{3x+4}+x)](x+\sqrt{5x-4})}{[(x-\sqrt{5x-4})(x+\sqrt{5x-4})](\sqrt{3x+4}+x)} =$$

$$= \lim_{x\to 4}\frac{[(\sqrt{3x+4})^2-(x)^2](x+\sqrt{5x-4})}{[(x)^2-(\sqrt{5x-4})^2](\sqrt{3x+4}+x)} = \lim_{x\to 4}\frac{[(3x+4)-x^2](x+\sqrt{5x-4})}{[x^2-(5x-4)](\sqrt{3x+4}+x)} =$$

$$= \lim_{x\to 4}\frac{(-x^2+3x+4)(x+\sqrt{5x-4})}{(x^2-5x+4)(\sqrt{3x+4}+x)} \underset{\substack{\text{factorizar}\\ \text{polinomios}}}{=} \lim_{x\to 4}\frac{-(x-4)(x+1)(x+\sqrt{5x-4})}{(x-4)(x-1)(\sqrt{3x+4}+x)} \underset{\text{simplifica r}}{=}$$

$$= \lim_{x\to 4}\frac{-(x+1)(x+\sqrt{5x-4})}{(x-1)(\sqrt{3x+4}+x)} = \frac{-(4+1)(4+\sqrt{5(4)-4})}{(4-1)(\sqrt{3(4)+4}+4)} == \frac{-5\cdot 8}{3\cdot 8} = -\frac{5}{3}$$

39) $$\lim_{x\to+\infty}\frac{\sqrt{3x^2+1}}{2+5x} = \frac{+\infty}{+\infty} = \lim_{x\to+\infty}\frac{\sqrt{\frac{3x^2}{x^2}+\frac{1}{x^2}}}{\frac{2}{x}+\frac{5x}{x}} \lim_{x\to+\infty}\frac{\sqrt{3+\frac{1}{x^2}}}{\frac{2}{x}+5} = \frac{\sqrt{3+0}}{0+5} = \frac{\sqrt{3}}{5}$$

40) $$\lim_{x\to-\infty}\frac{\sqrt{x^2-3x+1}}{2+5x} = \lim_{x\to+\infty}\frac{\sqrt{(-x)^2-3(-x)+1}}{2+5(-x)} = \lim_{x\to+\infty}\frac{\sqrt{x^2+3x+1}}{2-5x} = \frac{+\infty}{-\infty}(\text{I}) = \lim_{x\to+\infty}\frac{\sqrt{x^2}}{-5x} =$$

$$= \lim_{x\to+\infty}\frac{\sqrt{\frac{x^2}{x^2}+\frac{3x}{x^2}+\frac{1}{x^2}}}{\frac{2}{x}-\frac{5x}{x}} = \lim_{x\to+\infty}\frac{\sqrt{1+\frac{3}{x}+\frac{1}{x^2}}}{\frac{2}{x}-5} = \frac{\sqrt{1+0+0}}{0-5} = -\frac{1}{5}$$

<u>Recuerda</u>
$$\lim_{x\to-\infty} f(x) = \lim_{x\to+\infty} f(-x)$$

41) $$\lim_{x\to+\infty}\frac{\sqrt[3]{x^7-2x+4}}{3x^2-1} = \frac{+\infty}{+\infty}(\text{I}) = \lim_{x\to+\infty}\frac{\sqrt[3]{\frac{x^7}{x^7}-\frac{2x}{x^7}+\frac{4}{x^7}}}{\frac{3x^2}{x^{7/3}}-\frac{1}{x^{7/3}}} \lim_{x\to+\infty}\frac{\sqrt[3]{1-\frac{2}{x^6}+\frac{4}{x^7}}}{\frac{3}{x^{1/3}}-\frac{1}{x^{7/3}}} = \frac{\sqrt[3]{1-0+0}}{0-0} = \frac{1}{0} = +\infty$$

<u>Otra forma</u>

$$\lim_{x\to+\infty}\frac{\sqrt[3]{x^7-2x+4}}{3x^2-1} = \frac{+\infty}{+\infty}(\text{I}) = \lim_{x\to+\infty}\frac{\sqrt[3]{x^7}}{3x^2} = \lim_{x\to+\infty}\frac{x^{\frac{7}{3}}}{x^2} \underset{\text{grado N>grado D}}{=} +\infty$$

42) $$\lim_{x\to-\infty}\frac{\sqrt[3]{x^7-2x+4}}{3x^2-1} = \lim_{x\to+\infty}\frac{\sqrt[3]{(-x)^7-2(-x)+4}}{3(-x)^2-1} = \lim_{x\to+\infty}\frac{\sqrt[3]{-x^7+2x+4}}{3x^2-1} = \frac{-\infty}{+\infty}(\text{I}) =$$

$$= \lim_{x\to+\infty}\frac{\sqrt[3]{-\frac{x^7}{x^7}+\frac{2x}{x^7}+\frac{4}{x^7}}}{\frac{3x^2}{x^{7/3}}-\frac{1}{x^{7/3}}} \lim_{x\to+\infty}\frac{\sqrt[3]{-1+\frac{2}{x^6}+\frac{4}{x^7}}}{\frac{3}{x^{1/3}}-\frac{1}{x^{7/3}}} = \frac{\sqrt[3]{-1+0+0}}{0-0} = \frac{-1}{0} = -\infty$$

Otra forma

$$\lim_{x\to-\infty}\frac{\sqrt[3]{x^7-2x+4}}{3x^2-1}=\lim_{x\to+\infty}\frac{\sqrt[3]{(-x)^7-2(-x)+4}}{3(-x)^2-1}=\lim_{x\to+\infty}\frac{\sqrt[3]{-x^7+2x+4}}{3x^2-1}=\frac{-\infty}{+\infty}(\text{I})=$$

$$=\lim_{x\to+\infty}\frac{\sqrt[3]{-x^7}}{3x^2}=\lim_{x\to+\infty}\frac{-x^{\frac{7}{3}}}{x^2}\underset{\text{grado N>grado D}}{=}-\infty$$

43) $$\lim_{x\to+\infty}\left(\frac{\sqrt{x+9}-3}{x^2}\right)=\frac{+\infty}{+\infty}(\text{I})=\lim_{x\to+\infty}\left(\frac{\sqrt{\frac{x}{x^4}+\frac{9}{x^4}}-\frac{3}{x^2}}{\frac{x^2}{x^2}}\right)=\lim_{x\to+\infty}\left(\frac{\sqrt{\frac{1}{x^3}+\frac{9}{x^4}}-\frac{3}{x^2}}{1}\right)=\frac{\sqrt{0+0}-0}{1}=0$$

Otra forma

$$\lim_{x\to+\infty}\left(\frac{\sqrt{x+9}-3}{x^2}\right)=\frac{+\infty}{+\infty}(\text{I})=\lim_{x\to+\infty}\left(\frac{\sqrt{x}}{x^2}\right)=\lim_{x\to+\infty}\frac{x^{\frac{1}{2}}}{x^2}\underset{\text{grado N<grado D}}{=}0$$

44) $\lim\frac{x-1}{x^2+x-2}$ cuando $x\to+\infty$, $x\to0$, $x\to1$, $x\to3$

- $$\lim_{x\to+\infty}\frac{x-1}{x^2+x-2}=\frac{+\infty}{+\infty}(\text{I})=\lim_{x\to+\infty}\frac{\frac{x}{x^2}-\frac{1}{x^2}}{\frac{x^2}{x^2}+\frac{x}{x^2}-\frac{2}{x^2}}==\lim_{x\to+\infty}\frac{\frac{1}{x}-\frac{1}{x^2}}{1+\frac{1}{x}-\frac{2}{x^2}}=\frac{0-0}{1+0-0}=\frac{0}{1}=0$$
- $$\lim_{x\to0}\frac{x-1}{x^2+x-2}=\frac{-1}{-2}=\frac{1}{2}$$
- $$\lim_{x\to1}\frac{x-1}{x^2+x-2}=\frac{0}{0}(\text{I})=\lim_{x\to1}\frac{(x-1)}{(x-1)(x+2)}=\lim_{x\to1}\frac{1}{(x+2)}=\frac{1}{3}$$
- $$\lim_{x\to3}\frac{x-1}{x^2+x-2}=\frac{2}{10}=\frac{1}{5}$$

45) $\lim\frac{x^5+2x^4+4x^3+8x^2}{4x^4+12x^3+13x^2+6x+1}$ cuando $x\to+\infty$, $x\to0$, $x\to-1$, $x\to-3$

- $$\lim_{x\to+\infty}\frac{x^5+2x^4+4x^3+8x^2}{4x^4+12x^3+13x^2+6x+1}=\frac{+\infty}{+\infty}(\text{I})=\lim_{x\to+\infty}\frac{\frac{x^5}{x^5}+\frac{2x^4}{x^5}+\frac{4x^3}{x^5}+\frac{8x^2}{x^5}}{\frac{4x^4}{x^5}+\frac{12x^3}{x^5}+\frac{13x^2}{x^5}+\frac{6x}{x^5}+\frac{1}{x^5}}=$$

$$=\lim_{x\to+\infty}\frac{1+\frac{2}{x}+\frac{4}{x^2}+\frac{8}{x^3}}{\frac{4}{x}+\frac{12}{x^2}+\frac{13}{x^3}+\frac{6}{x^4}+\frac{1}{x^5}}=\frac{1}{0}=+\infty$$

- $\lim_{x\to 0}\dfrac{x^5+2x^4+4x^3+8x^2}{4x^4+12x^3+13x^2+6x+1}=\dfrac{0}{1}=0$

- $\lim_{x\to -1}\dfrac{x^5+2x^4+4x^3+8x^2}{4x^4+12x^3+13x^2+6x+1}=\dfrac{5}{0}=\begin{cases}\lim_{x\to -1^-}\dfrac{x^5+2x^4+4x^3+8x^2}{4x^4+12x^3+13x^2+6x+1}=\dfrac{5}{0^+}=+\infty\\ \lim_{x\to -1^+}\dfrac{x^5+2x^4+4x^3+8x^2}{4x^4+12x^3+13x^2+6x+1}=\dfrac{5}{0^+}=+\infty\end{cases}$

- $\lim_{x\to -3}\dfrac{x^5+2x^4+4x^3+8x^2}{4x^4+12x^3+13x^2+6x+1}=\dfrac{(-3)^5+2(-3)^4+4(-3)^3+8(-3)^2}{4(-3)^4+12(-3)^3+13(-3)^2+6(-3)+1}=\dfrac{-243+162-108+72}{324-324+117-18+1}=-\dfrac{117}{100}$

Ejercicio 3: Calcula los siguientes límites **(Indeterminación 1^{∞})**

1) $$\lim_{x\to1}\left(\frac{x^3+1}{x^2+1}\right)^{\frac{3}{x-1}}=1^{\infty}(\text{I})=\lim_{x\to1}\left(1+\underbrace{\frac{x^3+1}{x^2+1}-1}_{\text{operamos}}\right)^{\frac{3}{x-1}}=\lim_{x\to1}\left(1+\frac{x^3-x^2}{x^2+1}\right)^{\frac{3}{x-1}}=\lim_{x\to1}\left(1+\frac{1}{\frac{x^2+1}{x^3-x^2}}\right)^{\frac{3}{x-1}}=$$

$$=\lim_{x\to1}\left(1+\frac{1}{\frac{x^2+1}{x^3-x^2}}\right)^{\frac{x^2+1}{x^3-x^2}\cdot\frac{x^3-x^2}{x^2+1}\cdot\frac{3}{x-1}}=\lim_{x\to1}\left[\left(1+\frac{1}{\frac{x^2+1}{x^3-x^2}}\right)^{\frac{x^2+1}{x^3-x^2}}\right]^{\frac{x^3-x^2}{x^2+1}\cdot\frac{3}{x-1}}=$$

$$=\left[\lim_{x\to1}\left(1+\frac{1}{\frac{x^2+1}{x^3-x^2}}\right)^{\frac{x^2+1}{x^3-x^2}}\right]^{\lim\limits_{x\to1}\frac{x^3-x^2}{x^2+1}\cdot\frac{3}{x-1}}=e^{\lim\limits_{x\to1}\frac{x^3-x^2}{x^2+1}\cdot\frac{3}{x-1}}=e^{\lim\limits_{x\to1}\frac{x^2(x-1)}{x^2+1}\cdot\frac{3}{(x-1)}}=e^{\lim\limits_{x\to1}\frac{3x^2}{x^2+1}}=e^{\frac{3}{2}}=\sqrt{e^3}=e\sqrt{e}$$

2) $$\lim_{x\to2}\left(\frac{2x-1}{x+1}\right)^{\frac{x}{x-2}}=1^{\infty}(\text{I})=\lim_{x\to2}\left(1+\underbrace{\frac{2x-1}{x+1}-1}_{\text{operamos}}\right)^{\frac{x}{x-2}}=\lim_{x\to2}\left(1+\frac{x-2}{x+1}\right)^{\frac{x}{x-2}}=\lim_{x\to2}\left(1+\frac{1}{\frac{x+1}{x-2}}\right)^{\frac{x}{x-2}}=$$

$$=\lim_{x\to2}\left(1+\frac{1}{\frac{x+1}{x-2}}\right)^{\frac{x+1}{x-2}\cdot\frac{x-2}{x+1}\cdot\frac{x}{x-2}}=\lim_{x\to2}\left[\left(1+\frac{1}{\frac{x+1}{x-2}}\right)^{\frac{x+1}{x-2}}\right]^{\frac{x-2}{x+1}\cdot\frac{x}{x-2}}=$$

$$=\left[\lim_{x\to2}\left(1+\frac{1}{\frac{x+1}{x-2}}\right)^{\frac{x+1}{x-2}}\right]^{\lim\limits_{x\to2}\frac{x-2}{x+1}\cdot\frac{x}{x-2}}=e^{\lim\limits_{x\to2}\frac{x-2}{x+1}\cdot\frac{x}{x-2}}=e^{\lim\limits_{x\to2}\frac{x}{x+1}}=e^{\frac{2}{3}}=\sqrt[3]{e^2}$$

3) $$\lim_{x\to2}(x-1)^{\frac{1}{x-2}}=1^{\infty}(\text{I})=\lim_{x\to2}\left(1+\underbrace{x-1-1}_{\text{operamos}}\right)^{\frac{1}{x-2}}=\lim_{x\to2}(1+x-2)^{\frac{1}{x-2}}=\lim_{x\to2}\left(1+\frac{1}{\frac{1}{x-2}}\right)^{\frac{1}{x-2}}=$$

$$=\lim_{x\to2}\left(1+\frac{1}{\frac{1}{x-2}}\right)^{\frac{1}{x-2}}=e$$

4) $$\lim_{x\to+\infty}\left(\sqrt{\frac{x-2}{x+5}}\right)^{x}=\lim_{x\to+\infty}\left(\frac{x-2}{x+5}\right)^{\frac{x}{2}}=1^{\infty}(\mathrm{I})=\lim_{x\to+\infty}\left(1+\underbrace{\frac{x-2}{x+5}-1}_{\text{operamos}}\right)^{\frac{x}{2}}=\lim_{x\to+\infty}\left(1+\frac{-7}{x+5}\right)^{\frac{x}{2}}=\lim_{x\to+\infty}\left(1+\frac{1}{\frac{x+5}{-7}}\right)^{\frac{x}{2}}=$$

$$=\lim_{x\to+\infty}\left(1+\frac{1}{\frac{x+5}{-7}}\right)^{\frac{x+5}{-7}\cdot\frac{-7}{x+5}\cdot\frac{x}{2}}=\lim_{x\to+\infty}\left[\left(1+\frac{1}{\frac{x+5}{-7}}\right)^{\frac{x+5}{-7}}\right]^{\frac{-7}{x+5}\cdot\frac{x}{2}}=\left[\lim_{x\to+\infty}\left(1+\frac{1}{\frac{x+5}{-7}}\right)^{\frac{x+5}{-7}}\right]^{\lim\limits_{x\to+\infty}\frac{-7}{x+5}\cdot\frac{x}{2}}=$$

$$=e^{\lim\limits_{x\to+\infty}\frac{-7}{x+5}\cdot\frac{x}{2}}=e^{\lim\limits_{x\to+\infty}\frac{-7x}{2x+10}}=e^{-\frac{7}{2}}=\frac{1}{e^{7/2}}=\frac{1}{\sqrt{e^{7}}}=\frac{1}{e^{3}\sqrt{e}}=\frac{\sqrt{e}}{e^{4}}$$

5) $$\lim_{x\to+\infty}2^{\left(\frac{x^3-1}{x^3+x}\right)^{x^2+1}}=\left(\lim_{x\to+\infty}2\right)^{\lim\limits_{x\to+\infty}\left(\frac{x^3-1}{x^3+x}\right)^{x^2+1}}=2^{\lim\limits_{x\to+\infty}\left(\frac{x^3-1}{x^3+x}\right)^{x^2+1}}\underset{(*)}{=}2^{e^{-1}}=2^{\frac{1}{e}}=\sqrt[e]{2}$$

(*) $$\lim_{x\to+\infty}\left(\frac{x^3-1}{x^3+x}\right)^{x^2+1}=1^{\infty}(\mathrm{I})=\lim_{x\to+\infty}\left(1+\underbrace{\frac{x^3-1}{x^3+x}-1}_{\text{operamos}}\right)^{x^2+1}=\lim_{x\to+\infty}\left(1+\frac{-x-1}{x^3+x}\right)^{x^2+1}=\lim_{x\to+\infty}\left(1+\frac{1}{\frac{x^3+x}{-x-1}}\right)^{x^2+1}=$$

$$=\lim_{x\to+\infty}\left(1+\frac{1}{\frac{x^3+x}{-x-1}}\right)^{\frac{x^3+x}{-x-1}\cdot\frac{-x-1}{x^3+x}\cdot(x^2+1)}=\lim_{x\to+\infty}\left[\left(1+\frac{1}{\frac{x^3+x}{-x-1}}\right)^{\frac{x^3+x}{-x-1}}\right]^{\frac{-x-1}{x^3+x}\cdot(x^2+1)}=$$

$$=\left[\lim_{x\to+\infty}\left(1+\frac{1}{\frac{x^3+x}{-x-1}}\right)^{\frac{x^3+x}{-x-1}}\right]^{\lim\limits_{x\to+\infty}\frac{-x-1}{x^3+x}\cdot(x^2+1)}=e^{\lim\limits_{x\to+\infty}\frac{(-x-1)(x^2+1)}{x^3+x}}=e^{\lim\limits_{x\to+\infty}\frac{-x^3-x^2-x-1}{x^3+x}}\underset{(**)}{=}e^{-1}=\frac{1}{e}$$

(**) $$\lim_{x\to+\infty}\frac{-x^3-x^2-x-1}{x^3+x}=\frac{-\infty}{+\infty}=(\mathrm{I})=\lim_{x\to+\infty}\frac{\frac{-x^3}{x^3}-\frac{x^2}{x^3}-\frac{x}{x^3}-\frac{1}{x^3}}{\frac{x^3}{x^3}+\frac{x}{x^3}}=\lim_{x\to+\infty}\frac{-1-\frac{1}{x}-\frac{1}{x^2}-\frac{1}{x^3}}{1+\frac{1}{x^2}}=$$

$$=\frac{-1-0-0-0}{1+0}=-1$$

6) $\lim\limits_{x\to 1} x^{\frac{2}{1-x}} = 1^{\infty}\,(\text{I}) = \lim\limits_{x\to 1}(1+x-1)^{\frac{2}{1-x}} = \lim\limits_{x\to 1}\left(1+\dfrac{1}{\dfrac{1}{x-1}}\right)^{\frac{2}{1-x}} = \lim\limits_{x\to 1}\left(1+\dfrac{1}{x-1}\right)^{\frac{1}{x-1}\cdot(x-1)\cdot\frac{2}{1-x}} =$

$= \left[\lim\limits_{x\to 1}\left(1+\dfrac{1}{x-1}\right)^{\frac{1}{x-1}}\right]^{\lim\limits_{x\to 1}(x-1)\cdot\frac{2}{1-x}} = e^{\lim\limits_{x\to 1}\frac{2(x-1)}{1-x}} = e^{\lim\limits_{x\to 1}\frac{-2(1-x)}{1-x}} = e^{\lim\limits_{x\to 1}(-2)} = e^{-2} = \dfrac{1}{e^2}$

7) $\lim\limits_{x\to 0}\sqrt[2x]{1-4x} = \lim\limits_{x\to 0}(1-4x)^{\frac{1}{2x}} = 1^{\infty}\,(\text{I}) = \lim\limits_{x\to 0}(1+(-4x))^{\frac{1}{2x}} = \lim\limits_{x\to 0}\left(1+\dfrac{1}{\dfrac{1}{-4x}}\right)^{\frac{1}{2x}} =$

$= \lim\limits_{x\to 0}\left(1+\dfrac{1}{\dfrac{1}{-4x}}\right)^{\frac{1}{-4x}\cdot(-4x)\cdot\frac{1}{2x}} = \left[\lim\limits_{x\to 0}\left(1+\dfrac{1}{\dfrac{1}{-4x}}\right)^{\frac{1}{-4x}}\right]^{\lim\limits_{x\to 0}(-4x)\cdot\frac{1}{2x}} = e^{\lim\limits_{x\to 0}(-2)} = e^{-2} = \dfrac{1}{e^2}$