$\boxed{\text{EJERCICIO 10}}$

14) $4\log x - \log\left(x^2 - \frac{4}{5}\right) = \log 5$

$$\log x^4 = \underline{\log 5 + \log\left(x^2 - \frac{4}{5}\right)}$$

$$\log x^4 = \log\left[5 \cdot \left(x^2 - \frac{4}{5}\right)\right]$$

$$x^4 = 5 \cdot \left(x^2 - \frac{4}{5}\right)$$

$$x^4 = 5x^2 - 4$$

$$\boxed{x^4 - 5x^2 + 4 = 0} \qquad \underline{\text{BICUADRADA}}$$

* Realizamos el cambio de variable $x^2 = t$

$$t^2 - 5t + 4 = 0$$

$$t = \frac{5 \pm \sqrt{25-16}}{2} = \frac{5 \pm 3}{2} \begin{cases} t = 4 \\ t = 1 \end{cases}$$

* Deshacemos el cambio de variable

* $t = 4 \Rightarrow x^2 = 4 \Rightarrow x = \pm\sqrt{4} \Rightarrow x = \pm 2$
* $t = 1 \Rightarrow x^2 = 1 \Rightarrow x = \pm\sqrt{1} \Rightarrow x = \pm 1$

COMPROBACIÓN

$x=-2$ $\quad \underbrace{4\log(-2)}_{\uparrow} - \log\left((-2)^2 - \frac{4}{5}\right) = \log 5$

no existe $\Rightarrow$ $x=-2$ no es solución

$x=-1$ $\quad \underbrace{4\cdot\log(-1)}_{\uparrow} - \log\left((-1)^2 - \frac{4}{5}\right) = \log 5$

no existe $\Rightarrow$ $x=-1$ no es solución

$x=2$ $\quad 4\log 2 - \log\left(2^2 - \frac{4}{5}\right) = \log 5$

$$\log 2^4 - \log\left(\frac{16}{5}\right) = \log 5$$

$$\log\left(\frac{2^4}{16/5}\right) = \log 5$$

$$\log 5 = \log 5 \checkmark$$

$x=1$ $\quad \underbrace{4\log 1}_{0} - \log\left(1^2 - \frac{4}{5}\right) = \log 5$

$$-\log\left(\frac{1}{5}\right) = \log 5$$

$$-\log 5^{-1} = \log 5$$

$$-(-1)\cdot\log 5 = \log 5$$

$$\log 5 = \log 5 \checkmark$$

SOLUCIONES $\boxed{x=1 \quad y \quad x=2}$

20) $\log_5 x^2 + \log_5 (2x-1) = 0$ $\qquad \log_a 1 = 0$

$\log_5 [x^2 \cdot (2x-1)] = \log_5 1$

$x^2 \cdot (2x-1) = 1$

$\boxed{2x^3 - x^2 - 1 = 0}$

I) <u>Factorizamos el polinomio</u> $P(x) = 2x^3 - x^2 - 1$

Posibles raíces enteras $= \text{Div}(-1) = \{\pm 1\}$

$$\begin{array}{c|cccc} & 2 & -1 & 0 & -1 \\ 1 & & 2 & 1 & 1 \\ \hline & 2 & 1 & 1 & \boxed{0} \end{array}$$

$\Rightarrow$ 1 es raíz y $(x-1)$ factor $\Rightarrow P(x) = (x-1) \cdot \underbrace{(2x^2+x+1)}_{(*)}$

$2x^2 + x + 1 = 0$

$x = \dfrac{-1 \pm \sqrt{1-8}}{4}$ $\nexists$ solución en $\mathbb{R}$ $\Rightarrow$ <u>$(2x^2+x+1)$ es irreducible</u>

Por tanto, $\boxed{P(x) = (x-1) \cdot (2x^2+x+1)}$

II) <u>La ecuación es equivalente a</u> $(x-1) \cdot (2x^2+x+1) = 0$

$\to x - 1 = 0 \Rightarrow \boxed{x = 1}$

$\to 2x^2 + x + 1 = 0$ $\nexists$ solución en $\mathbb{R}$

COMPROBACIÓN

$x=1$ $\quad \log_5 1^2 + \log_5 (2\cdot 1 - 1) = 0$

$$\log_5 1 + \log_5 1 = 0$$

$$0 + 0 = 0 \checkmark$$

SOLUCIÓN $\boxed{x = 1}$

26) $\log \sqrt{3x+1} + \log 5 = 1 + \log \sqrt{2x-3}$

$$\log \sqrt{3x+1} + \log 5 = \log 10 + \log \sqrt{2x-3}$$

$$\log \left[5 \cdot \sqrt{3x+1}\right] = \log \left[10 \cdot \sqrt{2x-3}\right]$$

$$5\sqrt{3x+1} = 10\sqrt{2x-3}$$

$:5 \downarrow$

$$\sqrt{3x+1} = 2\sqrt{2x-3}$$

$$\left(\sqrt{3x+1}\right)^2 = \left(2\sqrt{2x-3}\right)^2$$

$$3x+1 = 4(2x-3)$$

$$3x+1 = 8x-12$$

$$13 = 5x$$

$$\frac{13}{5} = x$$

COMPROBACIÓN

$$\log\sqrt{3\cdot\frac{13}{5}+1}+\log 5 = 1+\log\sqrt{2\cdot\frac{13}{5}-3}$$

$$\log\sqrt{\frac{39}{5}+1}+\log 5 = \log 10+\log\sqrt{\frac{26}{5}-3}$$

$$\log\sqrt{\frac{44}{5}}+\log 5 = \log 10+\log\sqrt{\frac{11}{5}}$$

$$\log\left(5\cdot\sqrt{\frac{44}{5}}\right) = \log\left(10\sqrt{\frac{11}{5}}\right)$$

extraer

$$\log\left(5\sqrt{\frac{2^2\cdot 11}{5}}\right) = \log\left(10\sqrt{\frac{11}{5}}\right)$$

$$\log\left(10\sqrt{\frac{11}{5}}\right) = \log\left(10\sqrt{\frac{11}{5}}\right) \checkmark$$

Solución $\boxed{x=\frac{13}{5}}$

31) $\dfrac{\log(x^2+8x)}{\log(2x+1)} = 2$

¡OJO!

$$\frac{\log A}{\log B} \neq \log\left(\frac{A}{B}\right)$$

$$\log(x^2+8x) = 2\log(2x+1)$$

$$\log(x^2+8x) = \log(2x+1)^2$$

$$x^2+8x = (2x+1)^2$$

$$x^2+8x = 4x^2+4x+1 \Rightarrow \boxed{3x^2-4x+1=0}$$

$$x = \frac{4 \pm \sqrt{16-12}}{6} = \frac{4\pm 2}{6} \quad \begin{cases} x = 1 \\ x = \frac{1}{3} \end{cases}$$

COMPROBACIÓN

$x=1$

$$\frac{\log(1^2+8\cdot 1)}{\log(2\cdot 1+1)} = 2$$

$$\frac{\log 9}{\log 3} = 2 \Rightarrow \frac{\log 3^2}{\log 3} = 2 \Rightarrow$$

$$\Rightarrow \frac{2\cdot \log 3}{\log 3} = 2 \Rightarrow 2 = 2 \checkmark$$

$x = 1/3$

$$\frac{\log\left((1/3)^2+8\cdot 1/3\right)}{\log\left(2\cdot(1/3)+1\right)} = 2 \Rightarrow \frac{\log(25/9)}{\log(5/3)} = 2 \Rightarrow$$

$$\Rightarrow \frac{\log(5/3)^2}{\log(5/3)} = 2 \Rightarrow \frac{2\cdot \cancel{\log(5/3)}}{\cancel{\log(5/3)}} = 2 \Rightarrow 2 = 2 \checkmark$$

SOLUCIONES $\boxed{x = 1 \ \text{ y } \ x = 1/3}$